THAI NGUYEN UNIVERSITY
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APPLICATION OF EXTENDED KALMAN FILTER (EKF) IN PREDICTIVE CONTROL FOR A CLASS OF NONLINEAR OBJECT

MAJOR: CONTROL AND AUTOMATION ENGINEERING

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SUMMARY OF DOCTORIAL (PhD) DISSERTATION IN ENGINEERING

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Published scientific works

1. Hoang Duc Quynh, Nguyen Thi Viet Huong and Nguyen Doan Phuoc (2013), “Identify 2-way suspension crane system state by discrete Kalman observer”, *Science and Technology Magazine, Thai Nguyen University*, volume 106 (issue 06), pp.15-21.
2. Hoang Duc Quynh and Nguyen Doan Phuoc (2013), “Build nonlinear model for level-temperature control system and analyze the system”, *Science and Technology Magazine, Thai Nguyen University*, volume 110 (issue 10), pp. 15-26.
3. Hoang Duc Quynh, Nguyen Doan Phuoc and Nguyen Quang Hung (2014), “Design model predictive controller for industrial valve”, *Military Scientific Research and Technology Magazine, Military Institute of Science and Technology*, Special issue of automation, pp.12-18.
4. Hoang Duc Quynh, Nguyen Dinh Hoa and Nguyen Doan Phuoc (2014), “Some design methods of model predictive controller for rapid opening valve”, *Science and Technology Magazine, Thai Nguyen University*, volume 122 (issue 08), pp.167-171.

   Hoang Duc Quynh, Nguyen Van Chi, Nguyen Nhu Hien and Nguyen Doan Phuoc (2016), “Application of extended Kalman filter in designing the output feedback model predictive controller for Rotary Inverted Pendulum, *Science and Technology Magazine*, *Thai Nguyen University*, volume 151 (issue 06), pp.185-191.

6. Hoang Duc Quynh, Nguyen Nhu Hien and Nguyen Doan Phuoc (2016), **“**Design the output feedback model predictive controller for sustainable control of three-phase asynchronous motor”, *Military Scientific Research and Technology Magazine, Military Institute of Science and Technology*, issue 44, pp. 13-22.
7. Nguyen Doan Phuoc, Nguyen Duc Anh, Vu Tien Thanh, Pham Van Hung and Hoang Duc Quynh (2016), “Robust output tracking control with constraints for nonlinear system base on piecewise linear quadratic optimization and its perspective for practical application”, *Workshop on Vietnamese – German Technology Cooperation and Cultural Exchange*, pp.57-67.

the same time affirmed the availability to the practice of the proposed method and completely suitable to the theoretical judgment.

Shortcomings and next research direction

There are three shortcomings of the dissertation and there is also the next research direction from the dissertation author in the future. Specifically as follows:

1) Although the good stable tracking quality of the output feedback model predictive controllers on the basis of using extended Kalman filter and the state feedback model predictive controller owning to the linearization of each nonlinear predictive model segment that has been affirmed through the simulation experiment with some nonlinear objects, but still lack of close theoretical proof for it. Therefore, these shortcomings will be one of the next research directions of the dissertation author.
2) From the result of Algorithm 2.5 and Algorithm 2.6 for the nonlinear object class (2.39), (2.43), the author realized that these two algorithms can completely extend for the input delay nonlinear object class $\underline{u}_{k-\tau}$. This is also the next research direction of the dissertation author in the future.
3) The nonlinear object class mentioned by the dissertation is discontinuous (4.1) has interference $\underline{\xi}_k$, $\underline{\varsigma}_k$ additivity in the model, meanwhile the industrial object always exists in the continuous form by the time. The continuous model quantization by the time to have respective discontinuous model to serve the controller design can not avoid the influence of model error for control quality. Thus, in the future, the author will continue to research and develop the control algorithms built by the dissertation in order to be able to apply directly to the continuous system with nonlinear spreading interference $\underline{\xi}$, $\underline{\varsigma}$ in the generalized model:

$$\begin{cases} \frac{d\underline{x}}{dt} = \underline{f}(\underline{x},\underline{u},\underline{\xi}) \\ \underline{y} = \underline{g}(\underline{x},\underline{u},\underline{\varsigma}) \end{cases} \tag{4.2}$$

to further improve the control quality in the industrial practice.

PREAMBLE

Necessity of the dissertation theme

MPC-Model Predictive Control, is also known in the name of RHC-Receding Horizon Control, a type of control based on optimized platform with high application nature in fact, especially for the processes with many complex variables, have binding condition for the control problem. This thing was proved through over 3000 successful applications of this technique in process control, chemistry industry, oil and gas, processing… However, if comparing to the quantity of successful applications for pure linear processes, as applying to the control of non-linear industrial objects, being impacted by interference, both inside the system and output of the system, the rate of application quantity of this technique is rather modest. This thing has many reasons, mainly are:

Firstly: the state variables of the non-linear process is impacted mostly by interference, even they cannot be measured accurately, to ensure to have a good control quality.

Secondly: with non-linear processes, as using directly the non-linear model for the work of input signal prediction, the predictive formula is very complex with the complexity increasing according to the rate of power level with the width of predictive window, meanwhile the smaller the predictive window is, the weaker the control quality will be.

Thirdly: with limited predictive window, the predictive forecast technique always requires to have locking function in objective function, because only like that, the stable quality is ensured. However, with non-linear process, the question that how locking function is chosen to be reasonable, up to now is still open.

They are reasons as well as basic difficulties stated above that show the necessary of the dissertation theme relating to the research, investment of model predictive controller of output feedback with high availability with non-linear objects in industry.

Objective and duty of the dissertation

To overcome the first difficulty, the dissertation puts forth the duty of using Kalman filter to observe the state of process, instead of using measurement sensor which is normally impossible for many state variables. For non-linear processes, it will be extended Kalman filter, abbreviated as EKF (Extended Kalman Filter).

With the second difficulty, the dissertation proposes to use linear predictive model instead of using directly the non-linear model of the output signal predictive process. Together with the use of this linear predictive model, the third difficulty will also be settled, because at that time, the objective function becomes purely a square function according to control signal; therefore, the corresponding suitable function, if it is necessary to supplement, according to the theory of Bellman function, will also be a square function.

Scope, object and method of research of the dissertation

To implement the duty required by the theme for industrial object and non-linear processes, the dissertation puts forth the research objective in the short term to develop Kalman filter and model predictive control for bilinear object, and then expand to generalized non-linear object. Besides, the dissertation will also research the quality of non-output feedback linear model predictive controller on the basis of combining jointly the state observation sets, here it is Kalman filter, together with state feedback model predictive controller with linear predictive mode of digitizing each section. Such controller will be called by the dissertation as output feedback model predictive controller according to the separation principle.

To implement the research duty and achieve the research objective of the theme, the Dissertation uses the research methods:

Theoretical research, simulation research, testing research.

Scientific and reality significance of the theme

Scientific significance

The idea of digitalized linear of each non-linear model section in service of the model predictive control work is not new, however, the different point in this dissertation is that the author will use limited predictive window instead of unlimited one as some works did. That

The applicability of the above two algorithms to the practice has been also tested simulation by the dissertation on the bilinear system by input signal (2.14), (2.15) in example 2.1 and example 2.2

and the gained simulation result confirmed the good quality of this observer.

2) Built the state feedback predictive control method of the nonlinear system on the basis of using each segment linear predictive model with the finite predictive window, specifically built algorithms:

   a) Algorithm 2.3 and Algorithm 2.4 for controlling the state feedback of the bilinear system.

   b) Algorithm 2.5 and Algorithm 2.6 for controlling the state feedback of the nonlinear system.

   The applicability of the above algorithms to the practice has also been tested simulation by the dissertation with: Inverted pendulum system and rotary inverted pendulum system.

   The gained simulation result confirmed the good quality of the nonlinear predictive controller using this each segment linear predictive model, right as the judgment from the theory.

3) Built the output feedback model predictive controller by separation principle on the basis of coupling Kalman state observer and the state feedback model predictive controller proposed by the dissertation. The details of working steps of this controller are shown by the dissertation in Algorithm 2.7 and its revised version for the bilinear system only.

   The applicability of the above algorithm to the practice was also tested simulation successfully by the dissertation on: Inverted pendulum system and rotary inverted pendulum system.

   The gained simulation result also affirmed the high availability of the method to the industrial practice.

4) Conducted the theoretical verification test on the actual model: rotary inverted pendulum at the Measuring – Control Library of Thai Nguyen University of Technology. The test result verified the correctness of the algorithms proposed in the dissertation, at

the simulation on the objects: inverted pendulum and rotary inverted pendulum.

3) Carry out the theoretical verification test on real model: The rotary inverted pendulum at the Measuring – Control Libratory of Thai Nguyen University of Technology has model given in formula (3.41). The test result verified the correctness of algorithms proposed in the dissertation. The obtained experiment results confirm the feasibility to the practice of the method as desired and completely suitable to the theoretical judgment.

CONCLUSION AND RECOMMENDATIONS

Implemented issues

The dissertation theme is related to the output feedback control problem for objects, have discontinuous nonlinear model:

$$\begin{cases} \underline{x}_{k+1} = \underline{f}(\underline{x}_k, \underline{u}_k) + \underline{\xi}_k \\ \underline{y}_k = \underline{g}(\underline{x}_k, \underline{u}_k) + \underline{\varsigma}_k \end{cases} \tag{4.2}$$

satisfy the binding condition $\underline{u}_k \in U$, so that output $\underline{y}_k$ of the stable tracking system obtains the preset sample value, in which system (4.1) also has impact interference both inside the system by $\underline{\xi}_k$ (process interference) and $\underline{\varsigma}_k$ at output signal (measuring interference).

To solve the above problem, the dissertation proposed the direction as using extended Kalman filter (EKF, UKF) for filtering interference, at the same time observing the system state to provide the observed state value for the state feedback model predictive controller, create the output feedback controller by separation principle.

With the direction as above, the dissertation gained the following results:

1) Re-presented extended Kalman filtering methods (EKF) and UKF for the nonlinear system in the detailed algorithm form. Furthermore, the dissertation also supplemented the linear Kalman application method (KF) for observing each nonlinear system segment by optimality principle. This additionally proposed method has been built in detail by the dissertation into:
    a) Algorithm 2.1 for observing the bilinear system state.
    b) Algorithm 2.2 for observing the nonlinear system state.

thing will create more ability for the controller to achieve the following properties:

It is possible to easily process the binding conditions owing to optimized algorithms.

It is possible to control output signal directly according to preset signal without moving to stable control problem as doing when using LQR optimal controller.

By proposing new technique in designing the non-linear model predictive controller, summarizing by algorithms which are feasible and easy to install, the dissertation has scientific theoretical significances as follows:

Affirming the quality and high application ability to the fact of the controllers (algorithms) to the control fact of industrial objects.

Contributing more applications of extended Kalman filters in the output feedback model predictive controllers designed according to the principle of linearizing each predictive model sections.

Proposing and proving the stable theorem, thereby affirming the absoluteness of the proposed model predictive controllers.

Reality significance

The demand on applying the knowledge of control science always exists in all production processes. Therefore, the initial objective of the dissertation is to apply new model predictive controllers to many objects in the industry. The dissertation met the reality demand stated above, with specific reality significances as follows:

Providing model predictive controllers (specifically by algorithms) to non-linear objects in the industry.

Designing and testing the quality of output feedback model predictive controllers on the basis of applying extended Kalman filters to the objects: inverted pendulum and rotary inverted pendulum.

Structure of the dissertation

The dissertation has structure of 3 chapters, is presented in 129 pages. After chapter 1, presenting about available results of output feedback model predictive control technique with separate analysis comment ideas of the author about each specific method, in chapter 2, the dissertation presents in detail the innovative techniques of the author to improve the application ability of this technique to non-linear objects in the industry. In chapter 3, the dissertation will prove the availability of these innovative proposals on some specific industrial objects through simulation testing and testing on real system. Finally, the author will summarize the basic results achieved by the dissertation, the shortcomings, overcome directions and the issues to be researched later for completion.

CHAPTER 1

OVERVIEW ON OUTPUT FEEDBACK MODEL PREDICTIVE CONTROL METHOD

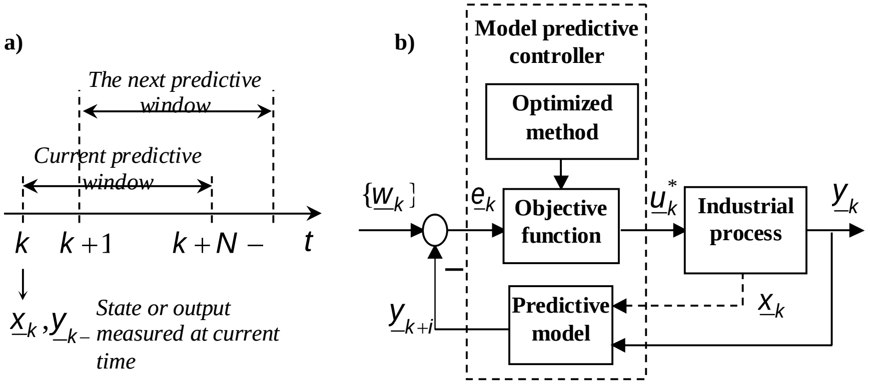

Figure 1.1: *Structure of model predictive control system*

1.1 Output feedback model predictive control of system with linear model

In term of actual application, the model predictive control was researched, developed very quickly over the past time. From the time of appearing the first model predictive controller introduced by engineers of Shell oil and gas Company in 1977, up to now, there are

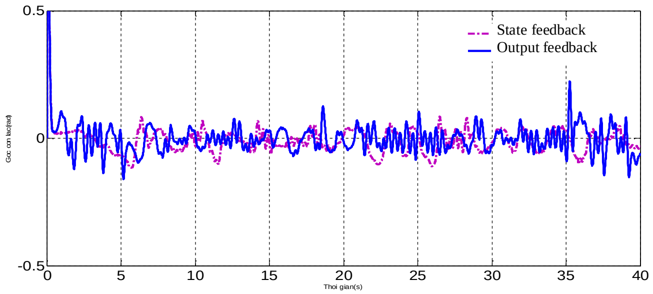

Figure 3.10: *Compare the controlled pendulum angle in two cases of output feedback and state feedback*

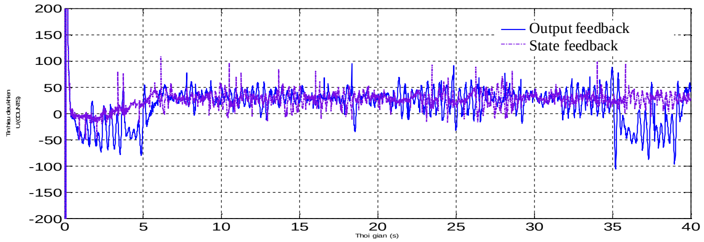

Figure 3.11: *Compare the control signal in two cases of output feedback and state feedback*

3.3 Conclusion of chapter 3

In chapter 3, the dissertation presented the following contents:

1) Apply the state feedback model predictive control of nonlinear system on the basis of using each segment linear predictive model proposed by the dissertation (Algorithm 2.3 - Algorithm 2.6) to the sample output signal tracking control for objects: inverted pendulum and rotary inverted pendulum for the control quality as desired.

2) Output feedback model predictive controller by separation principle on the basis of coupling Kalman state observer and state feedback model predictive controller proposed by the dissertation. The details of working steps of this controller have been shown by the dissertation in Algorithm 2.7. The working quality is good, meets the desired requirements of this output feedback controller was also affirmed by the dissertation through

3.2.3.2 Test result

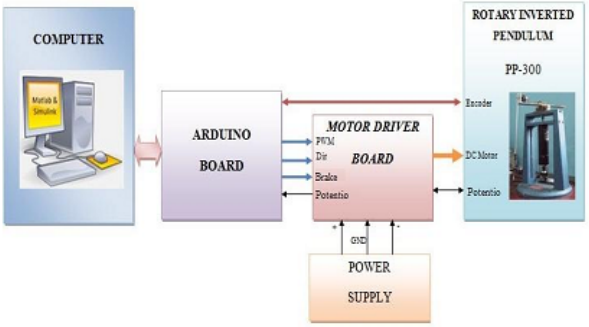

Figure 3.8: *Testing equipment connection diagram*

Figure 3.9: *Picture at the test table when controlling the pendulum in vertical position upward (pendulum angle tracking zero value) by output feedback model predictive controller under Algorithm 2.7*

quite many different versions of output feedback model predictive controller introduced, they affirm the position in applying to controlling many different industrial objects. However, they are mainly linear objects. These methods include:

- MAC (Model Algorithmic Control).
- DMC (Dynamic Matrix Control)
- GPC (Generalized Predictive Control).
- State feedback linear model predictive control.

In addition, in order to change a state feedback model predictive controller into output feedback model predictive controller, a very natural tendency is to use additionally the state observation set. The state observation set cared by the dissertation is Kalman filter. Therefore, in this overview part, the dissertation will also present additionally about the ability of output feedback model predictive control of linear system on the basis of combining the state feedback model predictive controller and linear Kalman filter, called briefly as output feedback controller according to the separation principle.

1.1.1 Method MAC (Model algorithmic control)

Algorithm 1.1 (MAC)

1.1.2 Method DMC (Dynamic matrix control)

Algorithm 1.2 (DMC)

1.1.3 Method GPC (Generalized predictive control)

Algorithm 1.3 (GPC)

1.1.4 Output feedback model predictive control according to separation principle for system with linear model

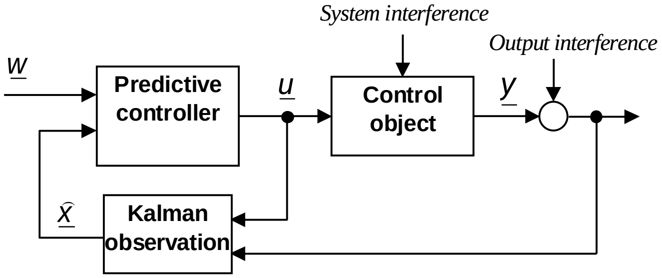

Figure 1.2: *Output feedback control according to separation principle*

State feedback model predictive control

Algorithm 1.4 ***(State feedback model predictive control of linear system)***

Observing the state of linear system with Kalman filter

Algorithm 1.5 (KF)

1.2 Method of output feedback model predictive control for system with non-linear model

1.2.1 State feedback model predictive control

Algorithm 1.6 ***(State feedback model predictive control of non-linear system)***

1.2.2 EKF-extended Kalman filter

A) EKF type 1: Linearizing by transformation of variable

Algorithm 1.7 ***(EKF type 1)***

B) EKF type 2: Linearizing surrounding predetermined orbit

Algorithm 1.8 ***(EKF type 2)***

C) EKF type 3: Linearizing the calculation steps inside Algorithm 1.5

Algorithm 1.9 ***(EKF type 3)***

1.2.3 UKF - Unscented Kalman Filter

Algorithm 1.10 ***(UKF)***

1.2.4 Output feedback model predictive control according to separation principle with non-linear Kalman filter

1.3 Some works of domestic and foreign authors in recent time researching about output feedback model predictive control

1.4 Research orientation of the dissertation

1.5 Conclusion of chapter 1

In chapter 1, the dissertation stated the overview of output feedback model predictive control methods, including simple direct output

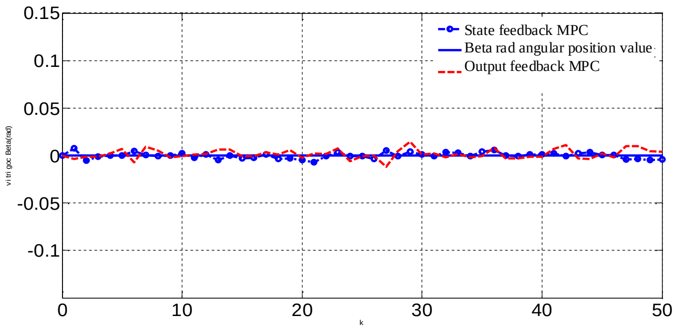

Figure 3.6: *Simulation result of output feedback controller compared with state feedback MPC for the rotary inverted pendulum (the output is the pendulum angular position by axis z)*

3.2.3 System description and test result

Figure 3.7: *Test model of rotary inverted pendulum Kri PP-300 at the Measuring – Control Laboratory, Thai Nguyen University of Technology*

3.1.3 Output feedback control by separation principle

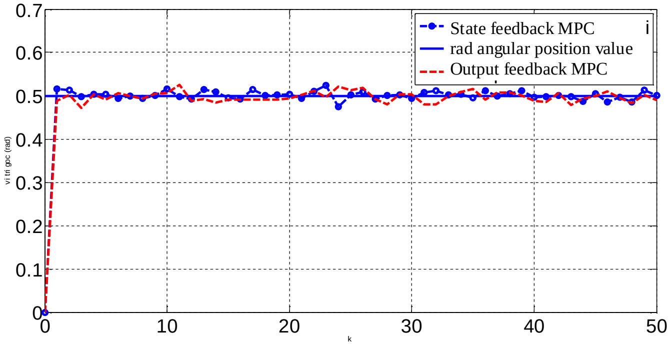

Figure 3.5: *Actual crab angle* $y_2 = \varphi$ *compared with set crab angle obtained owning to the output feedback model predictive controller when having system interference and output interference in white noise form*

3.2 Verify the quality on the test model of rotary inverted pendulum

3.2.1 Mathematical model of the rotary inverted pendulum

Starting from Lagrange equations that describe the movement of rotary inverted pendulum, equations that describe the direct current motor, after change of mathematics, we have mathematical model of the rotary inverted pendulum when the pendulum is in vertical position as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = K_1 x_2 - K_2 x_4 + K_3 + K_4 u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = K_5 x_2 - K_6 x_4 + K_7 + K_8 u \end{cases} \tag{3.46}$$

3.2.2 Simulation result of output feedback controller for the rotary inverted pendulum

feedback methods such as MAC, DMC, GPC (Algorithm 1.1 - Algorithm 1.3), mainly for SISO system, and indirect output feedback methods, or called as output feedback according to separation principle, also used for MIMO system.

All output control methods stated above are mainly built for linear system (Algorithm 1.4), and for non-linear system, it is only direction. The reason is that state feedback model predictive control algorithm (Algorithm 1.6) is very difficult to install and if it is possible to install, its calculation error is unavoidable because of high non-linear nature of objective function (1.36) calculated according to control signal $\underline{\boldsymbol{u}}$ determined according to formulas (1.34) and (1.35). Because the objective of the dissertation is to use Kalman filter as a state observation set, serve the problem of non-linear model predictive control by output feedback according to separation principle, thus in chapter 1, the dissertation presented briefly the content about linear Kalman filter (KF - Algorithm 1.5) as well as its extended forms (EKF -Algorithm 1.7 - Algorithm 1.9) and UKF (Algorithm 1.10) for non-linear system. Each extended type of Kalman filter has separate advantages and disadvantages and it should be applied for a class of special non-linear systems.

Through summarizing the domestic and foreign research results, analyzing the limitations which need further researching, the author also puts forth the research orientation of the dissertation in the ending part of the chapter.

On the basis of the analysis result about installation ability with little success of Algorithm 1.6 used for model predictive control of non-linear control, thus in chapter 2, the dissertation will propose another state feedback model predictive control method with higher feasibility, it can be used for MIMO non-linear system in general and bilinear system in particular. However, together with this non-linear state feedback model predictive control method proposed newly, the extended Kalman filter will also be innovated more suitably in order to be compatible to such method in output feedback control according to separation principle.

CHAPTER 2

KALMAN FILTER DESIGN FOR OBSERVING EACH SECTION OF STATE ACCORDING TO OPTIMAL PRINCIPLE AND APPLYING TO OUTPUT FEEDBACK MODEL PREDICTIVE CONTROL OF NON-LINEAR SYSTEM ACCORDING TO SEPARATION PRINCIPLE

2.1 Building Kalman observation set at each section for non-linear system

2.1.1 Kalman observation at each section for bilinear system

*Algorithm 2.1**: Kalman observation at each section for bilinear system** (2.5)**.*

1) Choose starting observation window $M \geq 2$ and weight matrix Λ according to (2.11).

2) Measure input and output values $\underline{u}_i$, $\underline{y}_i$, $i = 0,1, \dots, M$.

a) Determine all matrixes $\widehat{A}_i$, $\widehat{B}_i$, $\widehat{C}_i$, $\widehat{D}_i$, $i = 0,1, \dots, M$ from the model (2.5) of system according to the formula (2.6).

b) Calculate M of vectors $\underline{d}_i$, $i = 0,1, \dots, M-1$ according to (2.7) and $\underline{g}_i$, $i = 1,2, \dots, M$ according to (2.8). Building the vector $\underline{\boldsymbol{g}}$ and matrix G according to (2.9).

c) Calculate $\underline{x}_M^*$ according to (2.12). Assign $\widehat{\underline{x}}_M = \underline{x}_M^*$, $A_M = \widehat{A}_M$, $B_M = \widehat{B}_M$ and output $\widehat{\underline{x}}_M$ to make the state value to be observed of system (2.5) at the time M.

3) Assign $\underline{x}_M(+) = \widehat{\underline{x}}_M$ and choose $P_M(+)$ freely. Assign $k = M + 1$.

4) Measure $\underline{u}_k$, $\underline{y}_k$. Determine $C_k = C(\underline{u}_k, k)$, $D_k = D(\underline{u}_k, k)$.

5) Calculate:

$$\underline{x}_k(-) = A_{k-1}\underline{x}_{k-1}(+) + B_{k-1}\underline{u}_{k-1}$$

$$P_k(-) = A_{k-1}P_{k-1}(+)A_{k-1}^T + \Psi_{k-1}$$

$$K_k = P_k(-)C_k^T\left(C_kP_k(-)C_k^T + \Phi_k\right)^{-1}$$

$$P_k(+) = (I - K_kC_k)P_k(-)$$

$$\underline{x}_k(+) = \underline{x}_k(-) + K_k\left(\underline{y}_k - C_k\underline{x}_k(-) - D_k\underline{u}_k\right)$$

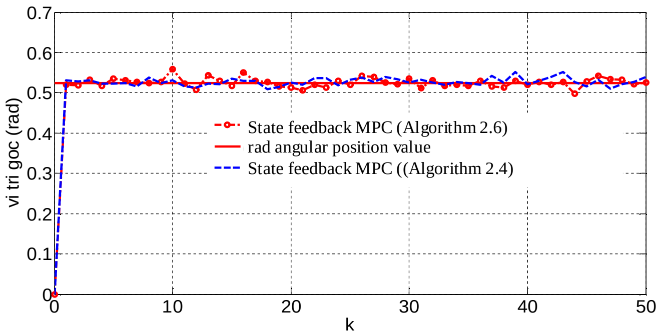

Figure 3.3: *Compare angular position control result at set value when using 2 controllers under Algorithm 2.4 and Algorithm 2.6*

Comment:

Through designing control and comparing the control quality with 2 controllers under Algorithm 2.4 (using bilinear model) and Algorithm 2.6 (using nonlinear model), may see immediately that, for the nonlinear objects with model that can be changed to the bilinear form, we should use Algorithm 2.4 to design control because the installation under Algorithm 2.4 is more simple and the calculation quantity is less than 2.6, at the same time the control quality is also better than using Algorithm 2.6, because in Algorithm 2.6 use approximation formula that may lead to model error. Therefore, for the nonlinear objects with model that can be changed to the bilinear form, we should use bilinear model to design control under Algorithm 2.4.

3.1.2 Observe the state with extended Kalman filter

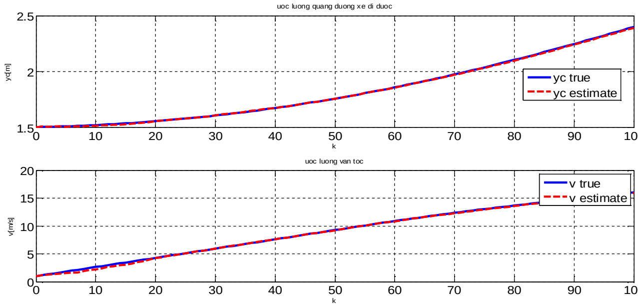

Figure 3.4: *The value of observed state x_3, x_4 is compared with the actual value when having system interference and output interference as Gauss interference*

CHAPTER 3

QUALITY EXPERIMENT AND VERIFICATION OF THE PROPOSED OUTPUT FEEDBACK MODEL PREDICTIVE CONTROLLER BY SEPARATION PRINCIPLE

3.1 Inverted pendulum control

3.1.1 State feedback predictive control

Use directly nonlinear model to design the state feedback model predictive controller of inverted pendulum under Algorithm 2.6

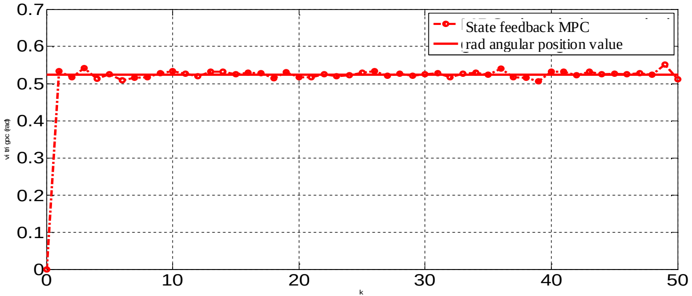

Figure 3.1: *Compare the actual crab angle with set crab angle before using directly nonlinear model to design the state feedback model predictive controller under Algorithm 2.6*

Use bilinear model to design the state feedback model predictive controller for inverted pendulum under Algorithm 2.4

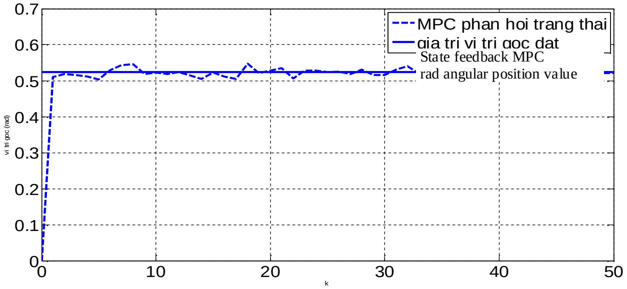

Figure 3.2: *Compare the actual crab angle with set crab angle before using bilinear model*

6) Output $\hat{\underline{x}}_k = \underline{x}_k(+)$ to make the state value to be observed of system (2.5) at the time k .

   Calculate $A_k = A(\underline{u}_k, k)$, $B_k = B(\underline{u}_k, k)$. Assign $k := k+1$ and return to 4).

Example 2.1 and Example 2.2: Illustration of Kalman observation set at each section

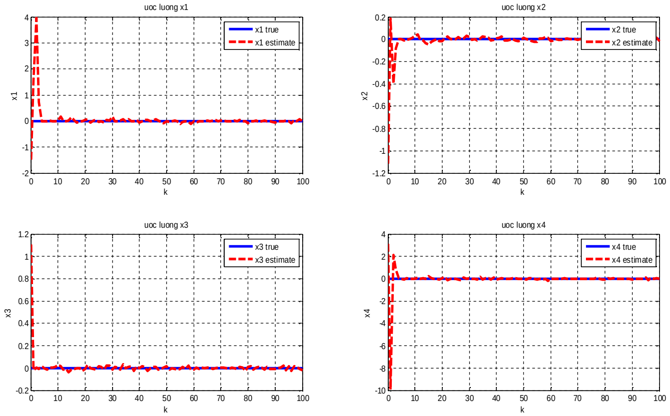

Figure 2.1: *Observed state and real state of the object as there are input interference and output interference with expected value* $\mu = 0$

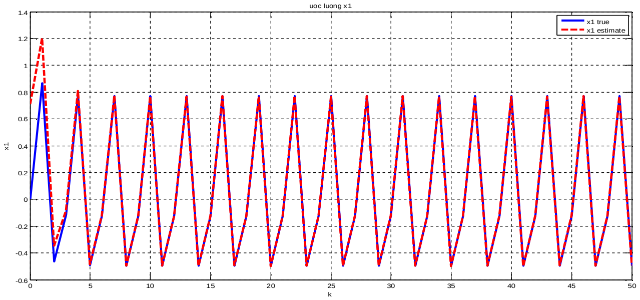

Figure 2.2: *State variable* $x_1[k]$ *as there are input interference and output interference with expected value* $\mu = 0$

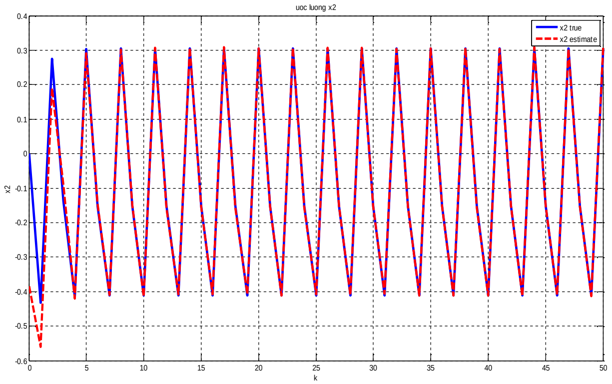

Figure 2.3: *State variable* $x_2[k]$ *as there are input interference and output interference with expected value* $\mu = 0$

2.1.2 Designing Kalman observation set at each section for non-linear system

State observation of non-linear system as knowing initial state

Algorithm 2.2a: Determining the state of non-linear system as having initial state $\underline{x}_0$.

Determining initial state approximation according to optimal standard

Algorithm 2.2b: Determine initial state $\underline{x}_0$.

Algorithm of observing each section of stage for non-linear system

Algorithm 2.2*: State observation of non-linear system.*

1) Choose observation window $M \geq 2$.
    a) Measure $M+1$ of input and output values $\underline{u}_i,\ \underline{y}_i,\ i = 0,1,\ \dots\ ,M$.
    b) Build the vectors of composite function $\underline{f}^i(\cdot),\ \underline{g} \circ \underline{f}^i(\cdot),\ i = 1,\ \dots\ ,M$ according to (2.22) and (2.23). Thence prepare objective function $J(\underline{\hat{x}}_0)$ according to (2.24), (2.25) and (2.26).
    c) Find optimal solution $\underline{\hat{x}}_0^*$ of the problem (2.26) owing to non-linear planning method.

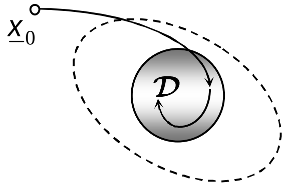

***Figure 2.6:** ISS stability of output feedback closed system by separation principle*

2.4 Conclusion of chapter 2

In chapter 2, the dissertation presented the following contents:

1) Extend linear Kalman Filter (KF) to apply to the nonlinear system on the basis of linearizing each nonlinear model segment of the system along the time axis and displacing on the time axis together with the predictive window of the model predictive control. Specifically, the dissertation has built:
    a) Algorithm 2.1 for observing the bilinear system state.
    b) Algorithm 2.2 for observing the nonlinear system state.

    The applicability of above two algorithms was also tested simulation by the dissertation on the bilinear object by input signal (2.14), (2.15) in example 2.1 and example 2.2.
2) The state feedback model predictive control of the nonlinear system on the basis of using the stage by stage linear predictive model, namely two algorithms:
    a) Algorithm 2.3 and Algorithm 2.4 for controlling the state feedback of bilinear system.
    b) Algorithm 2.5 or Algorithm 2.6 for controlling the state feedback of nonlinear system.
3) The output feedback model predictive controller by separation principle on the basis of coupling Kalman state observer and feedback model predictive controller proposed by the dissertation. The details of working steps of this controller were shown in Algorithm 2.7 by the dissertation.
4) A condition that is enough for the output model feedback controller is Algorithm 2.7 as ISS stability coefficient (actual stability).

1) Freely choose initial condition $\hat{x}_0 = \underline{x}_0(+)$ and $P_0(+)$. Measure $\underline{u}_0$, $\underline{y}_0$. Assign $k = 1$.

2) Measure $\underline{y}_k$. Choose two weight matrixes Λ_k, L_k symmetrically determining positive.

3) Calculate: $\underline{x}_k(-) = \underline{f}_k\left(\underline{x}_{k-1}(+), \underline{u}_{k-1}\right)$, G_k according to (1.47), where $\underline{g}(\underline{x}_k, \underline{u}_k) = C(\underline{x}_k)\underline{x}_k + D(\underline{x}_k)\underline{u}_k$ as the control object is system (2.44) or $\underline{g}(\underline{x}_k, \underline{u}_k) = C(\underline{x}_k)\underline{x}_k$ as the control object is (2.45). Calculate F_{k-1} according to (1.45) and:

$$P_k(-) = F_{k-1}P_{k-1}(+)F_{k-1}^T + \Psi_{k-1}$$

$$K_k = P_k(-)G_k^T(G_kP_k(-)G_k^T + \Phi_k)^{-1}$$

$$P_k(+) = \left(I - K_kG_k\right)P_k(-)$$

$$\underline{x}_k(+) = \underline{x}_k(-) + K_k\left(\underline{y}_k - G_k\underline{x}_k(-)\right)$$

4) Assign $\underline{x}_k = \underline{x}_k(+)$ and determine the matrixes A_k, B_k according to (2.40), C_k, D_k according to the formula similar to (2.42) but now it is amended to $C_k = C(\underline{x}_k)$, $D_k = D(\underline{x}_k)$, two matrixes E, F according to (2.45) if the control object is (2.52) or according to (2.50) if the control object is (2.53) and vector $\mathbf{z}$ according to (2.48) or according to (2.51).

5) Assign $k := k + 1$.

Calculate $\underline{u}_k$ according to (2.35) then enter the system control (2.52) or (2.53) in the period equal to the cycle of extracting the sample T_a then return to step 2).

2.3.2 ISS stability of output feedback controller

Theorem: *If the state feedback model predictive controller in Algorithm 2.3 - Algorithm 2.6, signed by (2.54), is Lipschitz function, non-linear system (2.1) has function vector $f(\underline{x}_k, \underline{u}_k)$ it is also Lipschitz function, the output feedback predictive controller of the dissertation (Algorithm 2.7) will make the system stabilize ISS.*

d) Determine the state values $\hat{\underline{x}}_i$, $i = 1, 2, \dots, M$ from $\hat{\underline{x}}_0^*$ according to (2.27).

2) Assign $\underline{e}_M(+) = \underline{0}$, $k = M + 1$.

Freely choose $P_M(+)$ and calculate $\hat{\underline{y}}_M = \underline{g}(\hat{\underline{x}}_M, \underline{u}_M)$.

3) Measure $\underline{u}_{k-1}, \underline{u}_k, \underline{y}_k$. Calculate:

$$A_k = \left.\frac{\partial \underline{f}}{\partial \underline{x}}\right|_{\hat{\underline{x}}_{k-1}, \underline{u}_{k-1}}, B_k = \left.\frac{\partial \underline{f}}{\partial \underline{u}}\right|_{\hat{\underline{x}}_{k-1}, \underline{u}_{k-1}},$$

$$C_k = \left.\frac{\partial \underline{g}}{\partial \underline{x}}\right|_{\hat{\underline{x}}_{k-1}, \underline{u}_{k-1}}, D_k = \left.\frac{\partial \underline{g}}{\partial \underline{u}}\right|_{\hat{\underline{x}}_{k-1}, \underline{u}_{k-1}} \text{ and } \Delta\underline{y}_k = \underline{y}_k - \hat{\underline{y}}_{k-1}$$

4) Calculate:

$$\underline{e}_k(-) = A_k\underline{e}_{k-1}(+) + B_k\underline{v}_{k-1}$$

$$P_k(-) = A_kP_{k-1}(+)A_k^T + \Psi_{k-1}$$

$$K_k = P_k(-)C_k^T(C_kP_k(-)C_k^T + \Phi_k)^{-1}$$

$$P_k(+) = \left(I - K_kC_k\right)P_k(-)$$

$$\underline{e}_k(+) = \underline{e}_k(-) + K_k\left(\Delta\underline{y}_k - C_k\underline{e}_k(-) - D_k\underline{v}_k\right)$$

5) Calculate $\hat{\underline{x}}_k = \underline{e}_k(+) + \hat{\underline{x}}_{k-1}$ and output as observed state value of the system.

6) Calculate $\hat{\underline{y}}_k = \underline{g}(\hat{\underline{x}}_k, \underline{u}_k)$. Assign $k := k + 1$ and return to 3).

2.2 State feedback model predictive control of non-linear system on the basis of using linear predictive model

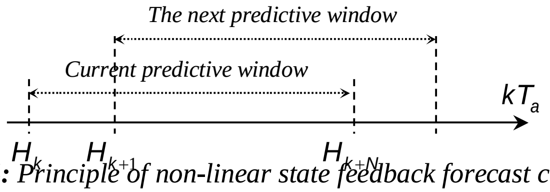

Figure 2.4: *Principle of non-linear state feedback forecast control on the basis of using linear predictive model*

2.2.1 Bilinear system control

Un-closed stoichiometric system control

*Algorithm 2.3**: Sample output signal adherence control for bilinear system** (2.28) **by state feedback model predictive controller.***

1) Assign $k = 0$. Choose the width of predictive window $N > 2$.
2) Measure the state $\underline{x}_k$ (or observe) and then determine the matrixes A_k, B_k, C_k, D_k according to (2.30), E, F according to (2.33), vector $\mathbf{z}$ according to (2.34). Choose two weight matrixes Λ_k, L_k symmetrically determining positive.
3) Calculate $\underline{u}_k$ according to (2.35) then enter the system control (2.28) in the period equal to the cycle of extracting the sample T_a.
4) Assign $k := k + 1$ then return to step 2).

Closed stoichiometric system control

*Algorithm 2.4**: Sample output signal adherence control for bilinear system** (2.36) **by state feedback model predictive controller.***

1) Assign $k = 0$. Choose the width of predictive window $N > 2$.
2) Measure the state $\underline{x}_k$ (or observe) and then determine the matrixes A_k, B_k, C_k, D_k according to (2.30), E, F (2.37), vector $\mathbf{z}$ according to (2.38). Choose two weight matrixes Λ_k, L_k symmetrically determining positive.
3) Calculate $\underline{u}_k$ according to (2.35) then enter the system control (2.36) in the period equal to the cycle of extracting the sample T_a.
4) Assign $k := k + 1$ then return to step 2).

2.2.2 Non-linear system control

Un-closed stoichiometric system control

*Algorithm 2.5**: Sample output signal adherence control for bilinear system** (2.39) **by state feedback model predictive controller.***

1) Assign $\underline{u}_{-1} = \underline{0}$, $\underline{x}_{-1} = \underline{0}$, $k = 0$. Choose the width of predictive window $N > 2$.
2) Measure the state $\underline{x}_k$ (or observe) and then determine the matrixes A_k, B_k according to (2.40), C_k, D_k according to (2.42), E, F according to (2.45) and vector $\mathbf{z}$ according to (2.48). Choose two weight matrixes Λ_k, L_k symmetrically determining positive.
3) Calculate $\underline{u}_k$ according to (2.35) then enter the system control (2.39) in the period equal to the cycle of extracting the sample T_a.
4) Assign $k := k + 1$ then return to step 2).

Closed stoichiometric system control

*Algorithm 2.6**: Sample output signal adherence control for bilinear system** (2.43) **by state feedback model predictive controller.***

2.3 Output feedback predictive control of non-linear system with extended Kalman filters

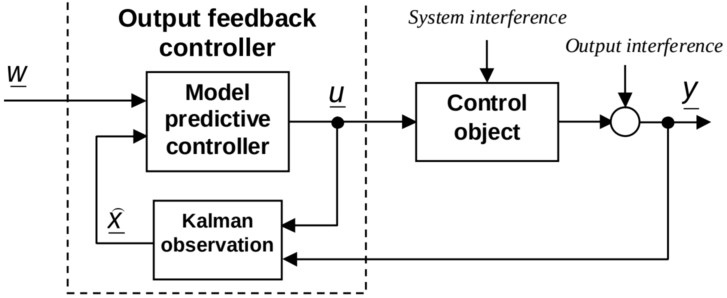

***Figure 2.5:** Structure of output feedback control system according to separation principle*

*Algorithm 2.7**: Output feedback predictive control according to separation principle for non-linear system** (2.44) **or** (2.45) **with Kalman EKF filter of type 3.***