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RESEARCHING MODEL ODER REDUCTION ALGORITHM AND APPLYING TO CONTROL PROBLEM

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SCIENTIFIC PAPERS

1. Cong Huu Nguyen, Kien Ngoc Vu, Hai Trung Do (2015), "Model reduction based on triangle realization with pole retention", *Applied Mathematical Sciences*, Vol. 9, 2015, No. 44, pp. 2187-2196, http://dx.doi.org/10.12988/ams.2015.5290
2. Vu Ngoc Kien, Đao Huy Du, Nguyen Huu Cong (2014), "*Model reduction in Schur basis with pole retention*", Journal of Science and Technology, No. 127(13), ISSN 1859 – 2171, pp. 101 – 106.
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INTRODUCTION

1. Introduction

2. The science and necessity of dissertation

In general technical technology and in particular control technology, mathematical model of the dynamic systems is used primarily for two purposes: simulation and control. In both of these purposes, we can face with complicated mathematical models, some of which can be higher-order ones / complicated mathematical models are more popular, some of which can be high-order ones such as weather prediction model in the Cohn study (1997), analyzing and designing Micro-Electro-Mechanical Systems (MEMS) in the Mukherjee study (2000), simulation of circuits in Chiprout study (1994), the higher-order optimal robust controller in the research of Trung (2012), Thanh (2008), the digital filter in the research of Zhang (2008),...

Theoretically, the complicated mathematical models, with higher-order will describe accurately the properties of dynamical systems - this is the main objective of mathematical model. However, the applying these models in reality will meet some disadvantages as following:

+ If the higher-order complicated model is model of object such as in studies of Chiprout (1994), Cohn (1997), Mukherjee (2000), it will increase the volume of calculations, leading to the rising of simulation time and might not meet the requirement on simulation time, as well as the learning model features; or if it meets the requirement on simulation time, the processing system requires high computation speed, and thus may result in high cost for hardware. Because of higher-order complicated models, memory capacity for data storage of these models need to be larger.

+ If the higher-order complex model is controller such as the model in the research of the Trung (2012) and Thanh (2008), it will increase the volume of calculations, so that the control systems might not meet the requirement on real-time control or if it satisfies the requirement on real-time control, it requires a high speed of hardware thus enhance the cost of control system or due to the complexity of controller, the control system will be easily damaged or reduced the reliability. In many cases, a control system with much higher-order complicated model might not be installed in devices such as automatic self-propelled equipment, the space robot... because of restricted space and volume of the device.

So that if there is a lower-order mathematical model which can describe relatively accurate the dynamical systems, the benefit will be as following:

+ Low-order models will reduce the volume of calculations that help calculation processing faster so it is easy to satisfy the requirements on time in simulations and in control.

+ Low-order models will reduce the volume of calculations, reduce data storage capacity so that it reduce the requirements on speed, memory capacity of hardware in simulation and control, respectively, the economic cost of system will be decreased or it will be effective in running old systems, and the small systems with lower hardware configuration (due to the restrictions on space and the volume). At the same time because of simple hardware structure (less element), the reliability of system will be raised.

Thus, low-order model has resolved harmoniously accuracy requirements of model with high speed calculating ability, the reliability of the system, lower economic costs. From this fact, seeking to identify lower-order model from higher-order original model satisfying several requirements is an urgent requirement and this is the main research inquiry/ research objectives of this dissertation.

In recent years, researching on model reduction for higher-order linear system has many results, but the existing algorithms still have disadvantages and need to be researched to further improve, especially the previous researching models for unstable higher-order linear system were very few and existed many disadvantages. Meanwhile, the higher-order linear system may be unstable, so it is necessary that order reduction algorithm must be able to reduce both stable system and unstable system. Thus, in this dissertation, the author focuses on researching systematically problem of model order reduction for linear system and then to proposes new linear model order reduction algorithm or completes existed linear model order reduction algorithm so that the algorithm can reduce both stable system and unstable system.

3. The objectives of the dissertation.

3.1. General Objectives

- Proposing new linear model order reduction algorithm or complete existed linear model order reduction algorithm in order that the algorithm can reduce both stable system and unstable system.

- Applying model order reduction algorithm in the field of control and automation such as reduced digital filter, reduced CD player model, reduced higher-order robust controller.

3.2. Specific objectives

- Proposing new standard measure to evaluate the importance of the pole in model order reduction algorithm. Thereby building new model order reduction algorithm for higher-order linear system and new model order reduction algorithm for unstable system; verify the effectiveness and correctness of the algorithm through some examples.

- Completing existed model order reduction algorithms in order that the algorithm better satisfies requirements of problem of higher-order linear unstable system; verify the effectiveness and correctness of the algorithm through some examples.

- Applying new model order reduction algorithm in one problem of the control field, namely the reduction of higher-order controller in two cases: the order reduction of higher-order controller of stable load angle system of synchronous generator (algorithms, simulation); the order reduction of higher-order controller of robust control system of self-balancing two-wheeled bicycle (including algorithms, simulation and experiment).

4. Research object, scope and methodology

5. Theoretical significance and practical significance

5.1. Theoretical significance

- Two model order reduction algorithms developed and completed in the this dissertation can be used to simplify the mathematical model described by a system of differential equations of n-order to a system of differential equations of r-order with $r<n$, but retain the essential features of the original model such as preserving the dominant poles, impotant Hankel singlar values and the input-output relationship of system is ensured so that the error between the original system and the order reduction one is not greater than a allowed value. Two algorithms help to supplement the system identification theory and the theory of control design in the field of control and electrical general.

- Applying two model order reduction algorithms to the order reduced problem of higher-order robust controller helps to gain lower-order controller but satisfies the requirements of robust control problem; this result helps to supplement the theory of lower-order robust control design in robust control problem.

5.2. Practical significance

- The results of study help to simplify higher-order controller or higher-order object model so that reduce the volume of calculations need to be processed (simple programming and installation), reduce data storage capacity so lessen requirements on processing speed, memory capacity of the hardware in simulation and control, respectively, decrease economic cost or effectively running the old system; the small structured systems (due to limited space and volume) with lower hardware configuration but still meet the quality demands. Simultaneously, when the hardware requirements in simulation and control are reduced or simple hardware structure (less elements), the reliability of the system will be raised.

- The result of study is reference for students, master students and PhD students who are interested in studying model order reduction and lower-order robust control design; able to add the stable and unstable automatic model order reduction in Matlab – Simulink toolbox.

6. Structure of dissertation.

CHAPTER 1. OVERVIEW OF MODEL ORDER REDUCTION

1.1. Model order reduction problem

A linear multiple input-multiple output system is given with continuous-time constant parameters described in space stated in the following equations:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} \end{aligned} \tag{1.1}$$

In which, $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \mathbb{R}^p$, $\mathbf{y} \in \mathbb{R}^q$, $\mathbf{A} \in \mathbb{R}^{nxn}$, $\mathbf{B} \in \mathbb{R}^{nxp}$, $\mathbf{C} \in \mathbb{R}^{qxn}$.

The goal of the order reduction problem with model described by (1.1) is to find models described by systems of equations:

$$\begin{aligned} \dot{\mathbf{x}}_r &= \mathbf{A}_r\mathbf{x}_r + \mathbf{B}_r\mathbf{u} \\ \mathbf{y}_r &= \mathbf{C}_r\mathbf{x}_r \end{aligned} \tag{1.2}$$

In which, $\mathbf{x}_r \in \mathbb{R}^r$, $\mathbf{u}_r \in \mathbb{R}^p$, $\mathbf{y}_r \in \mathbb{R}^q$, $\mathbf{A}_r \in \mathbb{R}^{rxr}$, $\mathbf{B}_r \in \mathbb{R}^{rxp}$, $\mathbf{C}_r \in \mathbb{R}^{qxr}$, with $r \ll n$ so that the model described by (1.2) can be replaced by the model described in (1.1), at the same time satisfying several requirements:

1. The order reduction error (ORE) should be small and equation is able to evaluate order reduction error;

2. The order reduction algorithm (ORA) should be calculated effectively and stably;

3. ORA should be automatically executed based on the upper bound formula of ORE;

4. The important properties of original systems like stability and passivity should be preserved

5. Suiting each particular requirement of each order reduction problem.

1.2. The order reduction researches in the world

1.2.1. The method base on Singular Perturbations Analysis (SPA)

1.2.2. The method base on Modal Analysis

1.2.3. The method base on Singular Value Decomposition (SVD)

1.2.4. The method base on Moment Matching (MM), or Krylov Methods

1.2.5. The method base on combination of SVD and MM

1.2.6. The other methods

1.3. The order reduction research in Vietnam

1.4. Problems need to be continuously researched on model order reduction

The dissertation of the author will focus on solving two problems:

The first problem is to propose an order reduction algorithm based on modal analysis method to overcome the disadvantages in the reseach of Du (2012), Rammos (2007), specifically, the algorithm will solve the following problems::

1. Building and expanding dominant evaluation standard of the poles which directly contact with ORE evaluation criterion to get small ORE; ;

2. Defining an upper bound formula of order reduction error;

3. With the same small ORE – the order of order reduction system is as small as possible;

4. Preserving dominant poles of the original system in order reduction system;

5. Reducing order of both stable system and unstable system;

6. Using popular mathematical tools, the less complex algorithms.

The second problem is to complete the ORA for unstable system under the second approach (the directly reduce order on stable system), namely the author will study to identify the upper bound formula of order reduction error of extending balanced truncation algorithm of Zilochian (1991) in order to automatically perform order reduction based on the upper bound formula of reduced order error.

1.5. Conclusion of chapter 1

In this chapter, the author researched and evaluated systematically problems of model order reduction for linear system, which showed that the proposed algorithms has their owned advantages and disadvantages, which should be applied to appropriate problem of the order reduction. In chorus, the proposed order reduction algorithms for linear system mainly applied to stable system, there are a few order reduction algorithms for unstable system with its advantages and disadvantages. Therefore, the dissertation of the author will be focused on solving two problems: Building a new model order reduction algorithm and completing the proposed algorithm of Zilochian's study (1991) to meet requirements of model order reduction for the linear system problem such as existing an upper bound formula of order reduction error, preserving dominant poles or Hankel's singular values, small order reduction error, simultaneously reducing order of both stable system and unstable system.

CHAPTER 2. BUILDING MODEL ORDER REDUCTION ALGORITHM

2.1. Introduction

2.2. The mathematical tools used in model order reduction algorithms

2.2.1. The matrix decomposition

2.2.1.1. The singular value decomposition (SVD)

2.2.1.2. The Schur decomposition

2.2.1.3. The Cholesky decomposition

2.2.2. The controllability and observability Gramian of stable systerm

2.3. New model order reduction algorithm for stable system

2.3.1. H_∞ dominance

$$\mathbf{A}_{\text{mod}} = \begin{bmatrix} \lambda_1 & & & 0 \\ & \cdot & & \\ & & \cdot & \\ 0 & & & \lambda_n \end{bmatrix}, \ \mathbf{B}_{\text{mod}} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix}, \ \mathbf{C}_{\text{mod}} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 \end{bmatrix}. \tag{2.7}$$

In Rammos (2007) research shown a definition of dominant pole as follows: *For given* $\mathbf{G}(s)$ *in diagonal realization (2.7), the pole* λ_i *of* $\mathbf{G}(s)$

is called dominant if its corresponding term $R_i := \dfrac{\|\mathbf{C}_i\mathbf{B}_i\|_2}{|\mathrm{Re}\,\lambda_i|}$ *is relatively large compared to others* $R_j\,(j \neq i)$. *The term* R_i *is called the dominance index of pole* λ_i.

The H_∞ norm error bound in modal truncation technique is given by Rammos (2007).

$$\|\mathbf{G}(s)-\mathbf{G}_{red}(s)\|_{H_\infty} \leq \sum_{\lambda_i \notin \Lambda_r} \frac{\|\mathbf{C}_i\mathbf{B}_i\|_2}{|\mathrm{Re}\,\lambda_i|} = \sum_{\lambda_i \notin \Lambda_r} R_i. \qquad (2.10)$$

2.3.2. Triangle realization

2.3.2.1. Algorithm to obtain triangle realization

Algorithm 2.3.2. Triangle realization

<u>**Input**</u>: Original system $(\mathbf{A},\ \mathbf{B},\ \mathbf{C})$ is described as (1.1)

<u>***Step 1***</u>: Compute Schur decomposition of $\mathbf{A}$: $\mathbf{A}=\mathbf{U}\boldsymbol{\Delta}\mathbf{U}^{\mathrm{T}}$, where $\mathbf{U}$ is unitary matrix and $\boldsymbol{\Delta}$ is upper triangle matrix.

<u>***Step 2***</u>: Compute observability Gramian $\mathbf{Q}$ came from Lyapunov equation

$$\boldsymbol{\Delta}\mathbf{Q}+\mathbf{Q}\boldsymbol{\Delta}+(\mathbf{CU})^{\mathrm{T}}(\mathbf{CU})=0 \qquad (2.11)$$

<u>***Step 3***</u>: Compute Cholesky factorization of observability Gramian $\mathbf{Q}$: $\mathbf{Q}=\mathbf{R}^{\mathrm{T}}\mathbf{R}$, where $\mathbf{R}$ is upper triangle matrix.

<u>***Step 4***</u>: Compute nonsingular transformation $\mathbf{T}=\mathbf{U}\mathbf{R}^{-1}$.

<u>***Step 5***</u> : Compute $(\tilde{\mathbf{A}},\ \tilde{\mathbf{B}},\ \tilde{\mathbf{C}})=(\mathbf{T}^{-1}\mathbf{AT},\ \mathbf{T}^{-1}\mathbf{B},\ \mathbf{CT})$.

<u>**Output**</u>: An equivalent system with realization $(\tilde{\mathbf{A}},\ \tilde{\mathbf{B}},\ \tilde{\mathbf{C}})$.

Definition 1. *The equivalent system* $(\tilde{\mathbf{A}},\ \tilde{\mathbf{B}},\ \tilde{\mathbf{C}})$ *in Algorithm 2.3.2 is said to be triangle realization.*

Lemma 1. *The equivalent system* $(\tilde{\mathbf{A}},\ \tilde{\mathbf{B}},\ \tilde{\mathbf{C}})$ *in Algorithm 2.3.2 has the following properties:*

(a) The matrix $\tilde{\mathbf{A}}$ *is upper triangle matrix,*

(b) The observability Gramian $\tilde{\mathbf{Q}}$ *is identity matrix; where* $\tilde{\mathbf{Q}}$ *is the solution of the following Lyapunov equation:*

$$\tilde{\mathbf{A}}^{\mathrm{T}}\tilde{\mathbf{Q}}+\tilde{\mathbf{Q}}\tilde{\mathbf{A}}+\tilde{\mathbf{C}}^{\mathrm{T}}\tilde{\mathbf{C}}=0, \qquad (2.12)$$

2.3.2.2. Factorization in triangle realization

We first partition $(\tilde{\mathbf{A}},\ \tilde{\mathbf{B}},\ \tilde{\mathbf{C}})$ thành dạng như sau:

$$\tilde{\mathbf{A}} = \begin{bmatrix} \tilde{\mathbf{A}}_{11} & \tilde{\mathbf{A}}_{12} \\ 0 & \tilde{\mathbf{A}}_{22} \end{bmatrix}, \ \tilde{\mathbf{B}} = \begin{bmatrix} \tilde{\mathbf{B}}_{1} \\ \tilde{\mathbf{B}}_{2} \end{bmatrix}, \ \tilde{\mathbf{C}} = \begin{bmatrix} \tilde{\mathbf{C}}_{1} & \tilde{\mathbf{C}}_{2} \end{bmatrix}.$$

Let $\mathbf{G}_1(s) := \tilde{\mathbf{C}}_1\left(s\mathbf{I} - \tilde{\mathbf{A}}_1\right)^{-1}\tilde{\mathbf{B}}_1$ and $\mathbf{G}_2(s) := \tilde{\mathbf{C}}_2\left(s\mathbf{I} - \tilde{\mathbf{A}}_2\right)^{-1}\tilde{\mathbf{B}}_2$ respectively be the trasfer functions of two subsystems.

Lemma 2. *With notations given as above we get that:*

$$\mathbf{G}(s) = \mathbf{G}_1(s) + \mathbf{V}(s)\mathbf{G}_2(s),$$

Where $\mathbf{V}(s) := \mathbf{I} - \tilde{\mathbf{C}}_1\left(s\mathbf{I} - \tilde{\mathbf{A}}_{11}\right)^{-1}\tilde{\mathbf{C}}_1^{\mathrm{T}}$. *Moreover,* $\mathbf{V}(s)$ *has property that*

$$\mathbf{V}^{\mathrm{T}}(-s)\mathbf{V}(s) = \mathbf{I}.$$

Assume that in triangle realization $\left(\tilde{\mathbf{A}}, \tilde{\mathbf{B}}, \tilde{\mathbf{C}}\right)$ has the following form

$$\tilde{\mathbf{A}} = \begin{bmatrix} \lambda_1 & * & * \\ & \ddots & * \\ 0 & & \lambda_n \end{bmatrix}, \tilde{\mathbf{B}} = \begin{bmatrix} \tilde{\mathbf{B}}_1 \\ \vdots \\ \tilde{\mathbf{B}}_n \end{bmatrix}, \tilde{\mathbf{C}} = \begin{bmatrix} \tilde{\mathbf{C}}_1 & \dots & \tilde{\mathbf{C}}_n \end{bmatrix}. \tag{2.19}$$

Let $\mathbf{g}_i(s) := \dfrac{\tilde{\mathbf{C}}_i\tilde{\mathbf{B}}_i}{s - \lambda_i};\ i = 1,\dots,n$ and $\mathbf{v}_i(s) := \mathbf{I} - \dfrac{\tilde{\mathbf{C}}_i\tilde{\mathbf{C}}_i^T}{s - \lambda_i};\ i = 1,\dots,n$.

Lemma 3. *With notations given as above we get that:*

$$\mathbf{G}(s) = \mathbf{g}_1(s) + \mathbf{v}_1(s)\mathbf{g}_2(s) + \mathbf{v}_1(s)\mathbf{v}_2(s)\mathbf{g}_3(s) + \dots + \mathbf{v}_1(s)\mathbf{v}_2(s)\dots\mathbf{v}_{n-1}(s)\mathbf{g}_n(s). \tag{2.20}$$

with $\mathbf{v}_i(s)$ *has property that*

$$\mathbf{v}_i^T(-s)\mathbf{v}_i(s) = \mathbf{I};\ i = 1,\dots,n-1.$$

2.3.2.3. Analysis of H_∞ - norm and H_2 – norm in triangle realization

Lemma 4. *Assume that* $\mathbf{G}(s)$ *is in triangle realization (2.19). Then*

$$\begin{aligned}\left\|\mathbf{G}(s)\right\|_{\mathrm{H}_\infty} &\le \left\|\mathbf{g}_1(s)\right\|_{\mathrm{H}_\infty} + \left\|\mathbf{g}_2(s)\right\|_{\mathrm{H}_\infty} + \dots + \left\|\mathbf{g}_n(s)\right\|_{\mathrm{H}_\infty} \\ &= \frac{\left\|\tilde{\mathbf{C}}_1\tilde{\mathbf{B}}_1\right\|_2}{\left|\operatorname{Re}\lambda_1\right|} + \frac{\left\|\tilde{\mathbf{C}}_2\tilde{\mathbf{B}}_2\right\|_2}{\left|\operatorname{Re}\lambda_2\right|} + \dots + \frac{\left\|\tilde{\mathbf{C}}_n\tilde{\mathbf{B}}_n\right\|_2}{\left|\operatorname{Re}\lambda_n\right|}.\end{aligned} \tag{2.21}$$

Lemma 5. *Assume that* $\mathbf{G}(s)$ *is in triangle realization (2.19). Then*

$$\begin{aligned}\left\|\mathbf{G}(s)\right\|_{\mathrm{H}_2} &\le \left\|\mathbf{g}_1(s)\right\|_{\mathrm{H}_2} + \left\|\mathbf{g}_2(s)\right\|_{\mathrm{H}_2} + \dots + \left\|\mathbf{g}_n(s)\right\|_{\mathrm{H}_2} \\ &= \sqrt{\operatorname{trace}\left(\tilde{\mathbf{B}}_1^{\mathrm{T}}\tilde{\mathbf{B}}_1\right)} + \sqrt{\operatorname{trace}\left(\tilde{\mathbf{B}}_2^{\mathrm{T}}\tilde{\mathbf{B}}_2\right)} + \dots + \sqrt{\operatorname{trace}\left(\tilde{\mathbf{B}}_{\mathrm{n}}^{\mathrm{T}}\tilde{\mathbf{B}}_n\right)}.\end{aligned} \tag{2.22}$$

Definition 2. *(H_∞ and H_2- dominant index) For given* $\mathbf{G}(s)$ *in triangle realization (2.19), the pole* λ_i *of* $\mathbf{G}(s)$ *is called* H_∞ *dominant if its* *corresponding term* $R_i := \dfrac{\left\|\tilde{\mathbf{C}}_i\tilde{\mathbf{B}}_i\right\|_2}{\left|\mathrm{Re}\lambda_i\right|}$ *is relatively large compared to others* $R_j, j \neq i$. *The term* R_i *is called the* H_∞ *dominant index of pole* λ_i. *Similarly, the pole* λ_i *of* $\mathbf{G}(s)$ *is called the* H_2*-dominant if its corresponding term* $S_i := \sqrt{\mathrm{trace}\left(\tilde{\mathbf{B}}_i^{\mathrm{T}}\tilde{\mathbf{B}}_i\right)}$ *is relatively large compared to others* $S_j, j \neq i$. *The term* S_i *is called the* H_2 *dominant index of pole* λ_i.

Definition 3. *(mixed-dominant index) The term* $J_i := \max\left\{R_i, S_i\right\}$ *is called mixed-dominant index coressponding to the pole* λ_i, *with* $i = 1,..,n$.

2.3.3. Model reduction based on triangle truncation

2.3.3.1. Analysis of H_∞ *and* H_2 *- norm upper error bound*

Theorem 1. *The error system satisfy the following properties :*

(a) $\left\|\mathbf{E}(s)\right\|_{\mathrm{H}_\infty} \leq \left\|\mathbf{G}_2(s)\right\|_{\mathrm{H}_\infty} \leq R_{r+1} + \ldots + R_n$,

(b) $\left\|\mathbf{E}(s)\right\|_{\mathrm{H}_2} \leq \left\|\mathbf{G}_2(s)\right\|_{\mathrm{H}_2} \leq S_{r+1} + \ldots + S_n$,

(c) $\max\left\{\left\|\mathbf{E}(\mathrm{s})\right\|_{\mathrm{H}_\infty} \left\|\mathbf{E}(\mathrm{s})\right\|_{\mathrm{H}_2}\right\} \leq \max\left\{\left\|\mathbf{G}_2(\mathrm{s})\right\|_{\mathrm{H}_\infty} \left\|\mathbf{G}_2(\mathrm{s})\right\|_{\mathrm{H}_2}\right\} \leq J_{r+1} + \ldots + J_n$,

where $\mathbf{G}_2(s) = \tilde{\mathbf{C}}_2\left(s\mathbf{I} - \tilde{\mathbf{A}}_{22}\right)^{-1}\tilde{\mathbf{B}}_2$ *is the subsystem of* $\mathbf{G}(s)$ *and* R_i, S_i, J_i *are corresponding the* H_∞, H_2 *and mixeddominant index of pole* λ_i, *with* $i = 1,\ldots,n$.

2.3.3.2. Re-ordering the poles by dominant indexes

Algorithm 2.3.3. (Re-ordering the poles by $\mathrm{H}_\infty, \mathrm{H}_2$ or mixed-dominant index)

Input: System $\left(\tilde{\mathbf{A}}, \tilde{\mathbf{B}}, \tilde{\mathbf{C}}\right)$ is the triangle realization of $\mathbf{G}(s)$, which is the output of Algorithm *2.3.2*.

Step 1: For each pole λ_i, with $i = 1,...,n$ compute its H_∞- dominant index $R_i = \dfrac{\left\|\tilde{\mathbf{C}}_i\tilde{\mathbf{B}}_i\right\|_2}{\left|\mathrm{Re}\lambda_i\right|}$ (or respectively, its H_2- dominant index $S_i = \sqrt{\mathrm{trace}\left(\tilde{\mathbf{B}}_i^{\mathrm{T}}\tilde{\mathbf{B}}_i\right)}$; or its mixed-dominant index $J_i = \max\left\{R_i, S_i\right\}$).

Step 2: Choose the largest H_∞- dominant index R_{i_1} (respectively for H_2- dominant index, mixed-index).

Step 3: Reorder the pole λ_{i_1} (and its conjugate $\overline{\lambda}_{i_1}$, if it appears) to the first position in the diagonal of $\tilde{\mathbf{A}}$ by unitary matrix $\mathbf{U}_1$:

$$\mathbf{U}_1^T\tilde{\mathbf{A}}\mathbf{U}_1 = \begin{bmatrix} \lambda_{i_1} & * & * & * & * \\ & \bar{\lambda}_{i_1} & * & * & * \\ & & * & * & * \\ & & & \ddots & \vdots \\ & & & & * \end{bmatrix}$$

Step 4: Compute new equivalent realization $\left(\mathbf{U}_1^T\tilde{\mathbf{A}}\mathbf{U}_1,\ \mathbf{U}_1^T\tilde{\mathbf{B}},\ \tilde{\mathbf{C}}\mathbf{U}_1\right)$.

Step 5: Remove two first rows and columns of $\left(\mathbf{U}_1^T\tilde{\mathbf{A}}\mathbf{U}_1,\ \mathbf{U}_1^T\tilde{\mathbf{B}},\ \tilde{\mathbf{C}}\mathbf{U}_1\right)$ to obtain a smaller realization $\left(\hat{\mathbf{A}},\ \hat{\mathbf{B}},\ \hat{\mathbf{C}}\right)$ with $n-2$ dimensions.

Step 6: Do the same procedure from Step 1 to Step 5 for smaller realization $\left(\hat{\mathbf{A}},\ \hat{\mathbf{B}},\ \hat{\mathbf{C}}\right)$ and continue this loop until all poles are re-ordered

Output: The equivalent system $\left(\breve{\mathbf{A}},\ \breve{\mathbf{B}},\ \breve{\mathbf{C}}\right)$ with the poles that are arranged in descending of H_∞- , H_2- and mixed-dominant index on the main diagonal of the upper – triangle matrix $\breve{\mathbf{A}}$

2.3.3.3. Reduced equivalent system

2.4. Example of reduced order high order stable system

2.4.1. Illustrative examples 1

2.4.2. Illustrative examples 2

2.5. New model order reduction algorithm for unstable system

2.5.1. Model order reduction algorithm for unstable system follow the indirect method (the first approach)

Algorithm 2.5.1. Model order reduction algorithm for unstable system follow the indirect method.

Input: The system $\left(\mathbf{A},\ \mathbf{B},\ \mathbf{C}\right)$ is described as (1.1) (Unstable system).

Step 1: Transform this system to the triangle system, we obtain the system that has the form:

$$\mathbf{A}_t = \begin{bmatrix} \mathbf{A}_{t11} & \mathbf{A}_{t12} \\ 0 & \mathbf{A}_{t22} \end{bmatrix}, \mathbf{B}_t = \begin{bmatrix} \mathbf{B}_{t1} \\ \mathbf{B}_{t2} \end{bmatrix}, \mathbf{C}_t = \left[\mathbf{C}_{t1} \quad \mathbf{C}_{t2}\right],$$

with $\mathbf{A}_{t11} \in \mathbb{R}^{mxm}$ (with m denotes the number of stable poles), $\mathbf{A}_{t12} \in \mathbb{R}^{mx(n-m)}$, $\mathbf{A}_{t22} \in \mathbb{R}^{(n-m)x(n-m)}$, $\mathbf{B}_{t1} \in \mathbb{R}^{mxp}$, $\mathbf{B}_{t2} \in \mathbb{R}^{(n-m)xp}$, $\mathbf{C}_{t1} \in \mathbb{R}^{qxm}$, $\mathbf{C}_{t2} \in \mathbb{R}^{qx(n-m)}$.

Step 2: Compute $\mathbf{S}$ came from Lyapunov equation:

$$\mathbf{A}_{t11}\mathbf{S} - \mathbf{S}\mathbf{A}_{t22} + \mathbf{A}_{t12} = 0.$$

Step 3: Define transition matrix

$$\mathbf{W} = \begin{bmatrix} \mathbf{I}_r & . & \mathbf{S} \\ \dots & . & \dots \\ 0 & . & \mathbf{I}_{n-r} \end{bmatrix},$$

with $\mathbf{I}_m$ and $\mathbf{I}_{n-m}$ are identity matrix of size mxm and $(n-m)x(n-m)$ respectively.

Step 4: Compute $(\mathbf{A}_d, \mathbf{B}_d, \mathbf{C}_d) = (\mathbf{W}^{-1}\mathbf{A}_t\mathbf{W}, \mathbf{W}^{-1}\mathbf{B}_t, \mathbf{C}_t\mathbf{W})$.

Step 5: Partitioning $(\mathbf{A}_d, \mathbf{B}_d, \mathbf{C}_d)$ as

$$\mathbf{A}_d = \begin{bmatrix} \mathbf{A}_{d11} & 0 \\ 0 & \mathbf{A}_{d22} \end{bmatrix}, \mathbf{B}_d = \begin{bmatrix} \mathbf{B}_{d1} \\ \mathbf{B}_{d2} \end{bmatrix}, \mathbf{C}_d = \begin{bmatrix} \mathbf{C}_{d1} & \mathbf{C}_{d2} \end{bmatrix},$$

with $\mathbf{A}_{d11} \in \mathbb{R}^{mxm}$, $\mathbf{A}_{d22} \in \mathbb{R}^{(n-m)x(n-m)}$

$\mathbf{B}_{d1} \in \mathbb{R}^{mxp}$, $\mathbf{B}_{d2} \in \mathbb{R}^{(n-m)xp}$, $\mathbf{C}_{d1} \in \mathbb{R}^{qxm}$, $\mathbf{C}_{d2} \in \mathbb{R}^{qx(n-m)}$,

Stable subsystem $(\mathbf{A}_{d11}, \mathbf{B}_{d1}, \mathbf{C}_{d1})$,

Unstable subsystem $(\mathbf{A}_{d22}, \mathbf{B}_{d2}, \mathbf{C}_{d2})$.

Step 6: Reduce stable subsystem $(\mathbf{A}_{d11}, \mathbf{B}_{d1}, \mathbf{C}_{d1})$ follow as *algorithm 2.3.2, 2.3.3 and 2.3.4* obtained reduced system $(\breve{\mathbf{A}}_{11}, \breve{\mathbf{B}}_1, \breve{\mathbf{C}}_1)$.

Output: A reduced system $\left((\breve{\mathbf{A}}_{11}, \breve{\mathbf{B}}_1, \breve{\mathbf{C}}_1) + (\mathbf{A}_{d22}, \mathbf{B}_{d2}, \mathbf{C}_{d2})\right)$.

Details algorithm 2.5.1 have been published in articles 1 and 2 of the author.

2.5.2. Model order reduction algorithm for unstable system follow the direct method (the second approach)

Definition 4. *The system (1.1) is called β - stable if* $\text{real}(\lambda(\mathbf{A})) < \beta$, *with β is a non-negative real numbers ($\beta \geq 0$).The set of continuous β - stable system is denoted by $\mathbb{C}_\beta$. The $\mathrm{H}_{\infty,\beta}$ - norm of $\mathbf{G}(s) \in \mathbb{C}_\beta$ is denoted by:*

$$\left\| \mathbf{G}(s) \right\|_{H_\infty,\beta} := \sup_{real(\lambda(\mathbf{A}))<\beta} \sigma_{\max}(\mathbf{G}(s))$$
$$= \sup_{\omega\in\mathbb{R}} \sigma_{\max}(\mathbf{G}(\beta + j\omega))$$

Where $\sigma_{\max}(\mathbf{G}(s))$ *is the largest singular value of* $\mathbf{G}(s)$.

Theorem 2: *For any continuous system* $\mathbf{G}(s) \in \mathbb{C}_\beta$ *represented by (1.1), we consider the system* $\mathbf{G}_\beta(s)$ *with realization* $\left(\mathbf{A}_\beta,\ \mathbf{B}_\beta,\ \mathbf{C}_\beta\right) = \left(\mathbf{A} - \beta\mathbf{I},\ \mathbf{B},\ \mathbf{C}\right)$. *Then, the following things hold::*

(i) $\mathbf{G}_\beta$ *is asymptotically stable,*

(ii) The H_∞ *- norm of* $\mathbf{G}_\beta(s)$ *is equal to the* $\mathrm{H}_{\infty,\beta}$ *- norm of* $\mathbf{G}(s)$, *...*

$$\left\| \mathbf{G}_\beta(s) \right\|_{\mathrm{H}_\infty} = \left\| \mathbf{G}(s) \right\|_{\mathrm{H}_{\infty,\beta}}.$$

Theorem 3: *Let* $\mathbf{G}(s) \in \mathbb{C}_\beta$ *and* $\hat{\mathbf{G}}_1(s)$ *is reduced order system obtained as in Algorithm 2.5.2. Then the following bound for the error system hold:*

$$\left\| \mathbf{G}(s) - \hat{\mathbf{G}}_1(s) \right\|_{H_{\infty,\beta}} \le 2\left(\sigma_{r+1} + \ldots + \sigma_n\right),$$

Where $\sigma_1,\ldots,\sigma_n$ *are the Hankel singular values of* $\mathbf{G}_\beta(s)$.

2.6. Example of reduced order of higher-order unstable system

2.6.1. Reduced order unstable linear system follow the indirect algorithm

2.6.2. Reduced order unstable linear system follow the direct algorithm

2.7. Conclusion of chapter 2

In this chapter, the author has gained the following contents:

1. Introducing some common mathematical tools in model order reduction.

2. Building a new model order reduction algorithm for stable system (***algorithm 2.3.2, algorithm 2.3.3***) based on the preserving the dominant poles of the original system in order reduction system. The most important new feature of the algorithm is given the H_∞- , H_2- and mixed-dominant index that used to assess the importance of the poles, the ability to arrange the poles follow the decreasing dominant index in the main diagonal of the upper triangular matrix $\mathbf{A}$ and given a upper bound formula of order reduction error for stable system. At the same time, author gave three new definitions, a new theorem and five new lemmas with the proof adequate.

3. Building a new model order reduction algorithm for unstable system follow the indirect method (***algorithm 2.5.1***) which is the extension of new algorithms for stable system (***algorithm 2.3.2, algorithm 2.3.3***)

based on preserving the dominant poles of the original system in order reduction system.

4. Giving one definition and two new theorem with the proof adequate to determine the upper bound formula of order reduction error ,thence complete extend balanced truncation algorithm for unstable system of Zilochian (1991) (***algorithm 2.5.2***).

5. Examples of order reduction in stable high order linear system (Digital filter model Zhang (2008), CD player models Rammos (2007)) and unstable high order linear system showed the correctness and effectiveness of the proposed algorithms.

CHAPTER 3. AN APPLICATION MODEL ORDER REDUCTION PROBLEM IN CONTROL

3.1. Introduction

3.2. Applying order reduction in controlling load angle problem of synchronous generator

In the study of Trung (2012), the author had designed robust controller RH_∞ to stabilize load angle of the synchronous generator when generator connected to the power grids and obtained a 28-order controller as follows:

$$\mathbf{R}(s)=\frac{\mathbf{N}(s)}{\mathbf{D}(s)}$$

$$\begin{aligned}\mathbf{N}(s)=&-0.004867s^{28}-0.7519s^{27}-58.8s^{26}-2526s^{25}-8.35.10^{4}s^{24}-2.128.10^{6}s^{23}\\&-4.383.10^{7}s^{22}-7.542.10^{8}s^{21}-1.108.10^{10}s^{20}-1.411.10^{11}\mathrm{s}^{19}-1.527.10^{12}s^{18}\\&-1.544.10^{13}s^{17}-1.341.10^{14}s^{16}-1.032e^{15}s^{15}-7.021.10^{15}s^{14}-4.211.10^{16}s^{13}\\&-2.213.10^{17}s^{12}-1.01.10^{18}s^{11}-3.954.10^{18}s^{10}-1.306.10^{19}s^{9}-3.564.10^{19}s^{8}\\&-7.845.10^{19}s^{7}-1.348.10^{20}s^{6}-1.723.10^{20}s^{5}-1.52.10^{20}s^{4}-8.162.10^{19}s^{3}\\&-1.984.10^{19}s^{2}+3.89.10^{16}s-125.2\end{aligned}$$

$$\begin{aligned}\mathbf{D}(s)=&\,5.25e^{-5}s^{28}+0.009786s^{27}+0.8675s^{26}+48.8s^{25}+1965s^{24}+6.056.10^{4}s^{23}\\&+1.49.10^{6}s^{22}-3.018.10^{7}s^{21}+5.14.10^{8}s^{20}+7.483.10^{9}s^{19}+9.425.10^{10}s^{18}\\&+1.035.10^{12}s^{17}+9.968.10^{12}s^{16}+8.432.10^{13}s^{15}+6.266.10^{14}s^{14}+4.079.10^{15}s^{13}\\&+2.314.10^{16}s^{12}+1.134.10^{17}s^{11}+4.74.10^{17}s^{10}+1.66.10^{18}s^{9}+4.762.10^{18}s^{8}\\&+1.085.10^{19}s^{7}+1.891.10^{19}s^{6}+2.399.10^{19}s^{5}+2.062.10^{19}s^{4}+1.065.10^{19}s^{3}\\&+2.479.10^{18}s^{2}-1.59.10^{4}s+2.945.10^{-11}\end{aligned}$$

3.2.1. Reduced order of controller follow the indirect order reduction algorithm

Table 3.1. Results of the order reduction of high order controller follow the indirect order reduction algorithm

Order	$\mathbf{R}_r(s)$
5	$\dfrac{-92.89s^5 - 2747s^4 - 2.202.10^4 s^3 - 1.515.10^5 s^2 - 3.974.10^5 s + 1495}{s^5 + 61.72s^4 + 1503s^3 + 1.944.10^4 s^2 + 1.167.10^5 s - 5.905.10^{-16}}$
4	$\dfrac{-92.89s^4 - 1042s^3 - 4767s^2 - 6.205.10^4 s + 85.25}{s^4 + 43.89s^3 + 717.7s^2 + 6651s - 3.366.10^{-17}}$

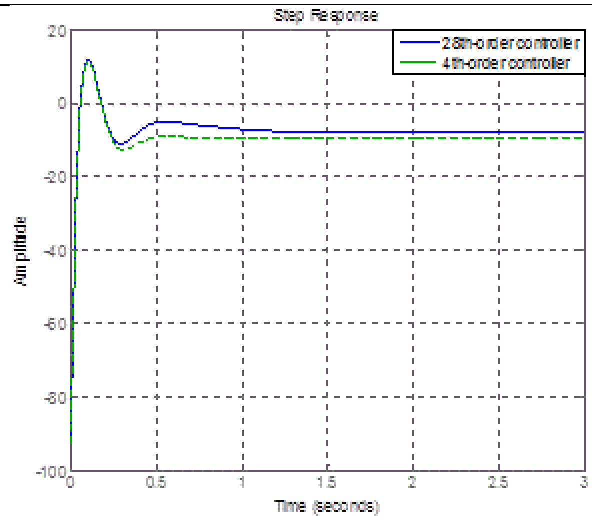


Fig 3.1. Step response of the original controller and 4th order controller

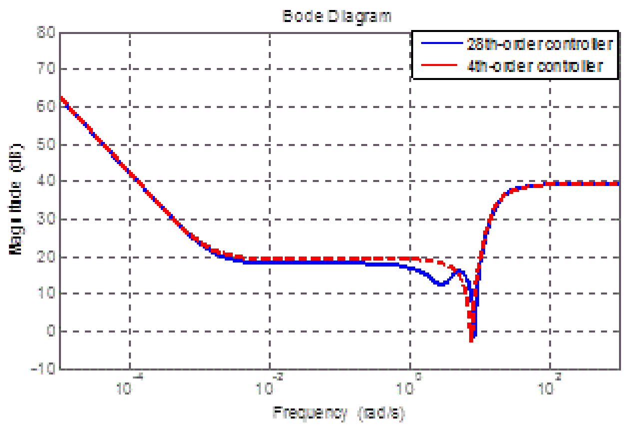


Fig 3.2. Bode diagram of the original controller and 4th order controller

3.2.2. Reduced order of controller follow the direct order reduction algorithm

Table 3.2. Results of the order reduction of high order controller follow the direct order reduction algorithm

Order	$\mathbf{R}_r(s)$
5	$\dfrac{-92.89s^5 - 438.1s^4 - 7570s^3 - 2.603.10^4 s^2 - 3.759.10^4 s - 1.26.10^4}{s^5 + 36.85s^4 + 557.6s^3 + 4799s^2 + 4428s + 1653}$

4	$\dfrac{-92.89s^4-424s^3-7535s^2-2.483.10^4s-3.513.10^4}{s^4+36.7s^3+552.5s^2+4720s+3923}$

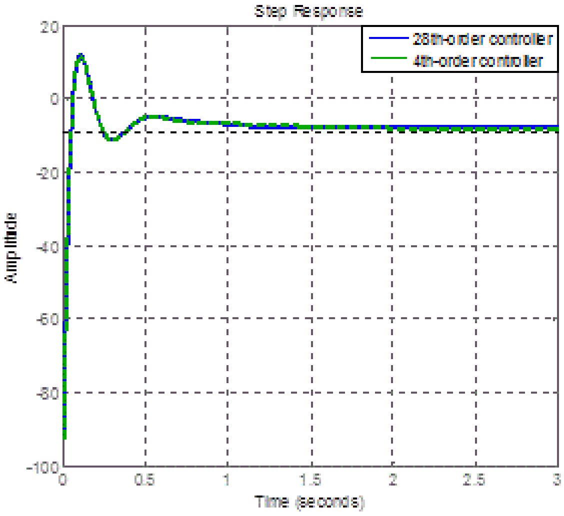


Fig 3.3. Step response of the original controller and 4^{th} order controller

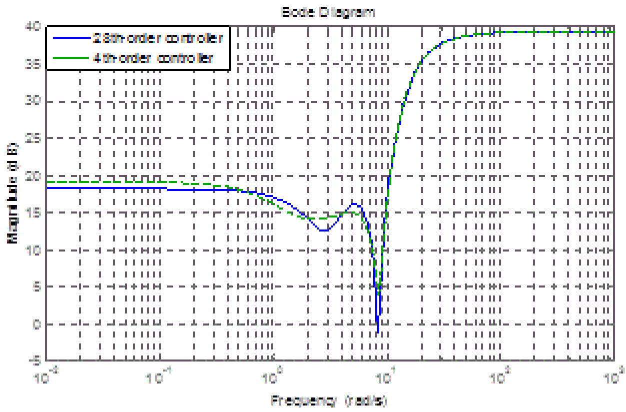


Fig 3.4. Bode diagram of the original controller and 4^{th} order controller

(*) Comparison of the results in the order reduction of 28^{th}-order controller with the results in the study of Trung (2012)

3.3. Applying order model reduction in balancing control problems of two-wheeled bicycle

3.3.1. Balance control problem of two-wheeled bicycle

The result of the design process, the author obtained completed model of self-balancing two-wheeled bicycle in Fig 3.5 as follows:

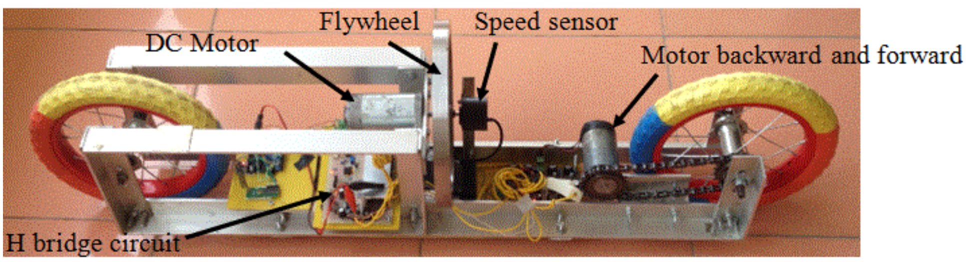


Fig 3.5. Complete model of self-balancing two-wheeled bicycle

Because of the uncertainty of the two-wheeled bicycle model, the author has designed robust control system RH_∞ for self-balancing two-wheeled bicycle shown in Appendix 10 and Appendix 11. The results, the author obtained a controller as follows:

$$\mathbf{R}(s) = \frac{\mathbf{H}(s)}{\mathbf{D}(s)}$$

With

$$\begin{aligned}\mathbf{H}(s) = &-2.23.10^{-7}s^{30} - 4.67.10^{-4}s^{29} - 0.266s^{28} - 22.96s^{27} - 1006s^{26} - 2.853.10^{4}s^{25}\\ &- 5.837.10^{5}s^{24} - 4.199.10^{11}s^{18} - 9.144.10^{6}s^{23} - 1.139.10^{8}s^{22} - 1.158.10^{9}s^{21}\\ &- 9.776.10^{9}s^{20} - 6.949.10^{10}s^{19} - 2.172.10^{12}s^{17} - 9.663.10^{12}s^{16} - 3.71.10^{13}s^{15}\\ &- 1.231.10^{14}s^{14} - 3.53.10^{14}s^{13} - 8.74.10^{14}s^{12} - 1.862.10^{15}s^{11} - 3.398.10^{15}s^{10}\\ &- 5.276.10^{15}s^{9} - 6.903.10^{15}s^{8} - 7.511.10^{15}s^{7} - 6.676.10^{15}s^{6} - 4.721.10^{15}s^{5}\\ &- 2.556.10^{15}s^{4} - 9.953.10^{14}s^{3} - 2.482.10^{14}s^{2} - 2.977.10^{13}s - 0.00439\end{aligned}$$

$$\begin{aligned}\mathbf{D}(s) = &\,4.971.10^{-14}s^{30} + 2.032.10^{-10}s^{29} + 2.663.10^{-7}s^{28} + 1.221.10^{-4}s^{27} + 9.72.10^{-3}s^{26}\\ &+ 0.3918s^{25} + 10.14s^{24} + 187.1s^{23} + 2612s^{22} + 2.862.10^{4}s^{21} + 1.088.10^{7}s^{18}\\ &+ 2.523.10^{5}s^{20} + 1.82.10^{6}s^{19} + 5.428.10^{7}s^{17} + 2.273.10^{8}s^{16} + 8.005.10^{8}s^{15}\\ &+ 2.372.10^{9}s^{14} + 5.9.10^{9}s^{13} + 1.225.10^{10}s^{12} + 2.107.10^{10}s^{11} + 2.962.10^{10}s^{10}\\ &+ 3.341.10^{10}s^{9} + 2.941.10^{10}s^{8} + 1.931.10^{10}s^{7} + 8.743.10^{9}s^{6} + 2.286.10^{9}s^{5}\\ &+ 1.519.10^{8}s^{4} - 5.226.10^{7}s^{3} + 3.6.10^{-6}s^{2} + 5.32.10^{-22}s\end{aligned}$$

3.3.2. Reduced order of robust controller follow the indirect order reduction algorithm

Table 3.4. Results of the order reduction of high order controller

Order	Reduced system $\mathbf{R}_r(s)$
5	$\dfrac{-4.485.10^{6}s^{5} - 6.804.10^{7}s^{4} - 4.123.10^{8}s^{3} - 1.235.10^{9}s^{2} - 1.816.10^{9}s - 1.09.10^{9}}{s^{5} + 2009s^{4} + 1.833.10^{4}s^{3} - 1913s^{2} + 2.165.10^{-13}s - 2.804.10^{-14}}$
4	$\dfrac{-4.485.10^{6}s^{4} - 2.65.10^{7}s^{3} - 1.141.10^{8}s^{2} - 1.833.10^{8}s - 1.176.10^{8}}{s^{4} + 2000s^{3} - 206.5s^{2} + 2.369.10^{-14}s - 3.026.10^{-15}}$

3.3.3. Reduced order of robust controller follow the direct order reduction algorithm

Table 3.6. Results of the order reduction of high order controller follow the extend balanced truncation algorithm

Order	Reduced system $\hat{\mathbf{R}}_1(s)$

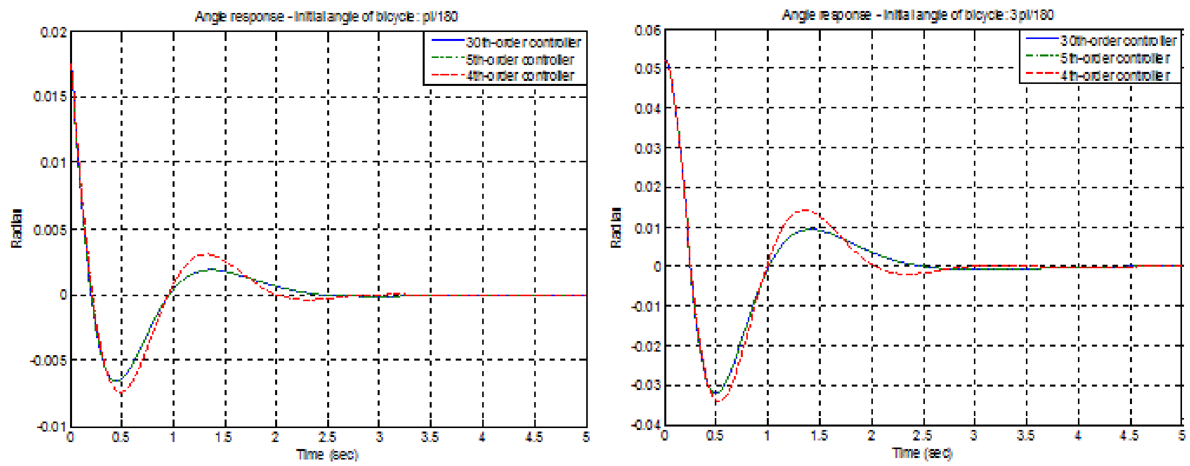
5	$\frac{-4.485.10^6 s^5 - 6.804.10^7 s^4 - 4.123.10^8 s^3 - 1.235.10^9 s^2 - 1.816.10^9 s - 1.09.10^9}{s^5 + 2009s^4 + 1.833.10^4 s^3 - 1913s^2 + 6.614.10^{-9} s - 8.44.10^{-10}}$
4	$\frac{-4.485.10^6 s^4 - 2.655.10^7 s^3 - 1.191.10^8 s^2 - 1.811.10^8 s - 1.182.10^8}{s^4 + 2000s^3 - 205.6s^2 - 0.1231s + 0.003463}$

3.3.4. Applying reduced order of controller to control balancing two-wheeled bicycle

3.3.4.1. Follow the indirect order reduction algorithm

Simulation results:

- When the parameters of the two-wheeled bicycle model are nominal (table 9.1 in Appendix 9) and the initial angle of the bicycle is $\theta = \left(\frac{\pi}{180} \div \frac{3\pi}{180}\right)(rad)$, the simulation results of balance control system of the two-wheeled bicycle are shown in Fig 3.7 as follows:



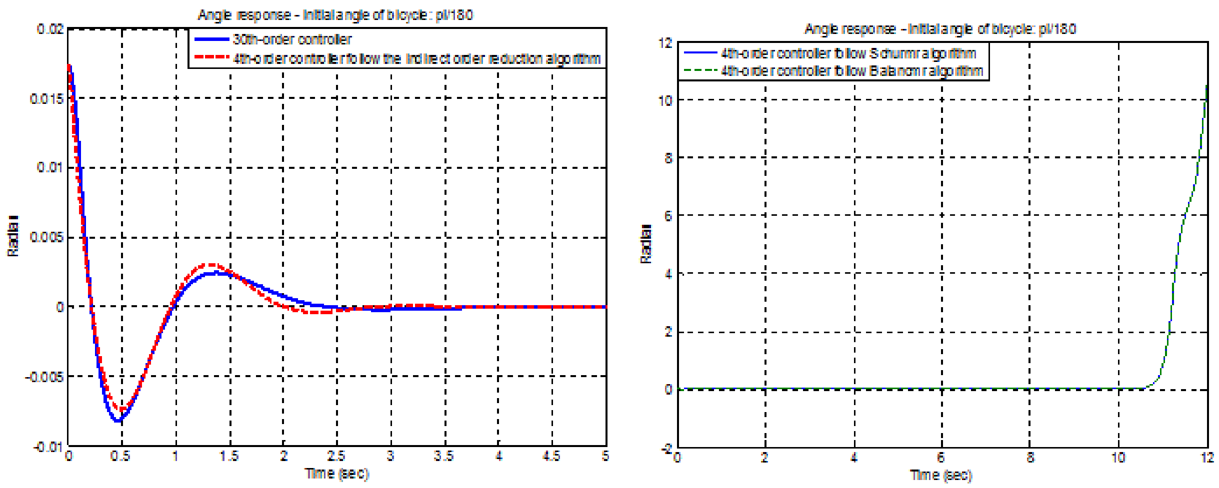
(a) (b)

Fig 3.7. Output response of the balanced control system of the two-wheeled bicycle used the original controller and 5^{th}-, 4^{th} – order controller

(*) Comparing balance control system of two-wheeled bicycle used the original controller, order reduction controller followed new order reduction method and order reduction controller followed other reduction methods.

Simulation results:

- When the parameters of the two-wheeled bicycle model are nominal (table 9.1 in Appendix 9) and the initial angle of the bicycle is $\theta = \frac{\pi}{180}(rad)$, the simulation results of balance control system of the two-wheeled bicycle are shown in Fig 3.9 as follows:



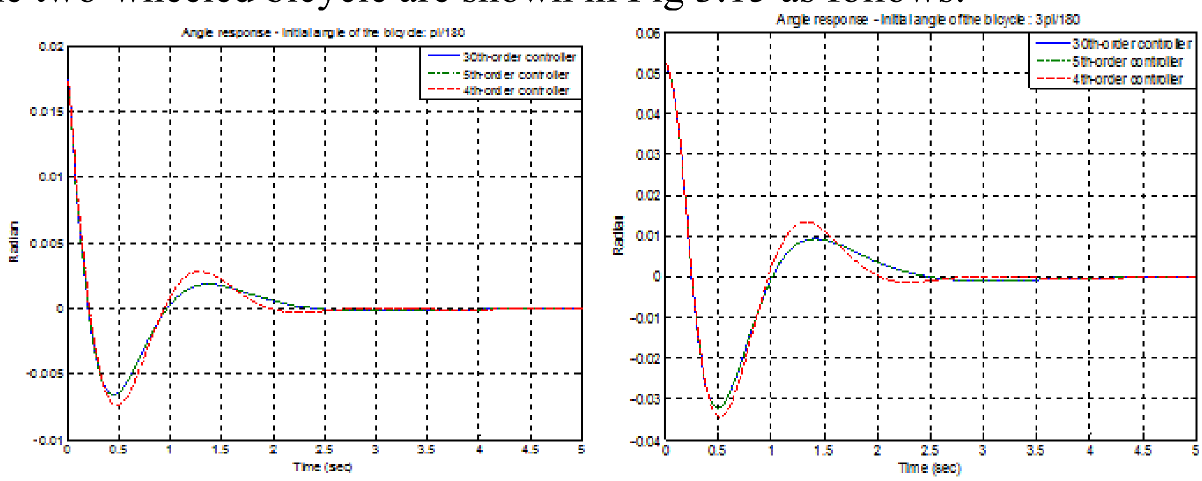
(a) (b)

Fig 3.9. Output response of the balance control system of the two-wheeled bicycle used the original controller and 4^{th} – order controller

3.3.4.2. Follow the direct order reduction algorithm

<u>Simulation results:</u>

- When the parameters of the two-wheeled bicycle model are nominal (table 9.1 in Appendix 9) and the initial angle of the bicycle is $\theta = \left(\dfrac{\pi}{180} \div \dfrac{3\pi}{180}\right)(rad)$, the simulation results of balance control system of the two-wheeled bicycle are shown in Fig 3.15 as follows:



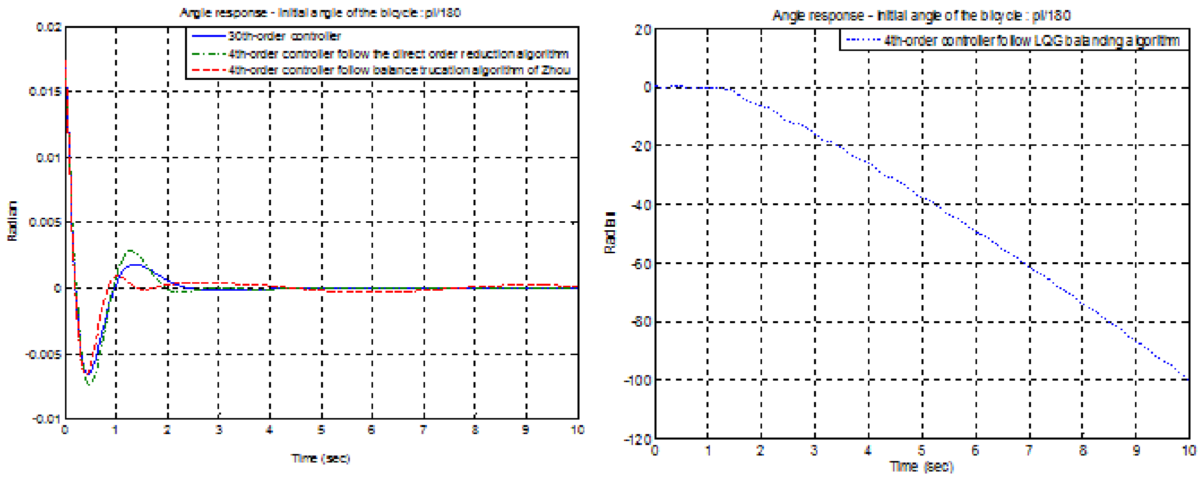
(a) (b)

Fig 3.15. Output response of the balance control system of the two-wheeled bicycle used the original controller and 5^{th}-, 4^{th} – order controller

(*) Comparing balance control system of two-wheeled bicycle used the original controller, order reduction controller followed direct order reduction method and order reduction controller followed other reduction methods.

<u>Simulation results:</u>

- When the parameters of the two-wheeled bicycle model are nominal (table 9.1 in Appendix 9) and the initial angle of the bicycle is $\theta = \frac{\pi}{180}(rad)$, the simulation results of balance control system of the two-wheeled bicycle are shown in Fig 3.117a and 3.17b as follows:



(a) (b)

Fig 3.17. Output response of the balance control system of the two-wheeled bicycle used the original controller and 4^{th} – order controller

3.4. Conclusion of chapter 3

In chapter 3, the author has achieved some results as following:

1. Applying the reduced-order algorithm proposed in chapter 2 to reduce the 28^{th}-order robust controller of stable load angle system of the synchronous generator in the study of Trung (2012),thus showed the 28^{th} –order controller can be replaced by the 4^{th} –order controller but still keep control system quality. On the other hand, the order of the controllers in this study is lower than the one in the research of Trung (2012), lower order controller helps control program simpler, better system control speed and real-time control ability, as well as reduction hardware system cost compared to the results of Trung study (2012).

2. Applying new indirect and direct order reduction algorithm to reduce 30^{th}-order robust controller of balance control system of two-wheeled bicycle showed that: Using 5^{th} – order, 4^{th}-order controller to control self-balancing two-wheeled bicycle (bicyle model is built in a laboratory) gives results of output response equivalent to using 30^{th}-order controller, thus the 30^{th}-order controller can totally be replaced by the 5^{th} -,

4^{th} – order controller to control the balance of two-wheeled bicycle but quality (simulation) of control system is still guaranteed.

3. Comparing the effectiveness of indirect order reduction algorithm with other ones in reducing order 30^{th} –order robust controller of balance control system of two-wheeled bicycle showed that: applying two-wheeled bicycel control system used 4^{th}-order reduction controller follow new algorithim can control balancing two-wheeled bicycel while follow the extend balance truncation algorithms of Moore (***balancmr***) and Schur (***Schunmr***) can not control the balance of bicycle.

4. Comparing the effectiveness of extend balance algorithm to other direct one in reducing order 30^{th}-order robust controller of balance control system of two-wheeled bicycel showed that: applying 4^{th}-order controller follow the extend balance truncation algorithm to control balancing two-wheeled bicycel gave better out-put response quality than LQG and Zhou algorithms.

5. Simulation results showed the correctness of two order reduction algorithms and the applied possibility to reduce order of high-order controller in robust controller problems.

CHAPTER 4. EXPERIMENTS

4.1. Experimental control system of self-balancing two-wheeled bicycle

Experimental control system of self-balancing two-wheeled bicycle is shown in Fig 4.16 as follows:

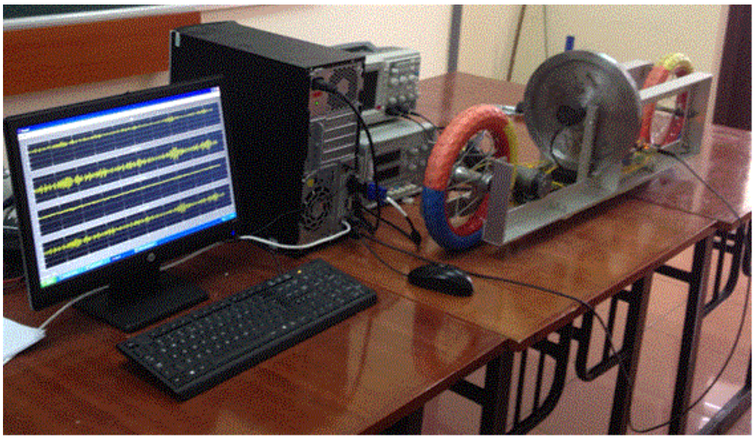


Fig 4.16. Experimental control table of self-balancing two-wheeled bicycle

4.2. Experimental results

- Operation of self-balancing two-wheeled bicycle when it doesn't carry the load is shown in Fig 4:17.

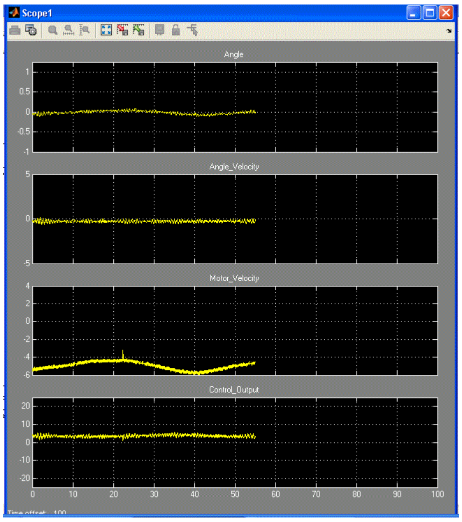


Fig 4.17. Response of self-balancing two-wheeled bicycle when it doesn't carry the load

- Operation of self-balancing two-wheeled bicycle when it is affected by an external force is shown in Fig 4.18 as follows:

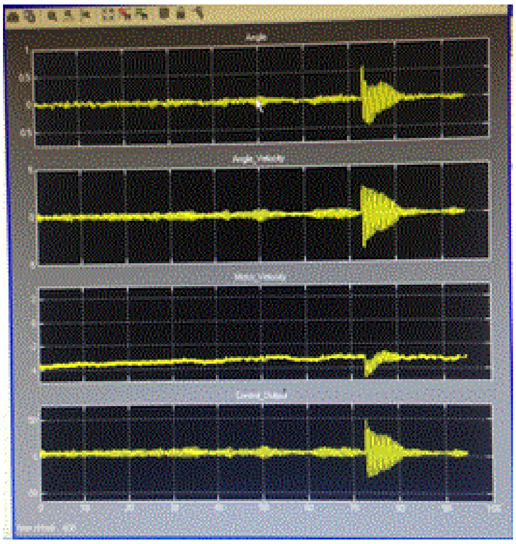


Fig 4.18. Response of self-balancing two-wheeled bicycle when it is affected by an external force

- Operation of self-balancing two-wheeled bicycle when it carries the load is shown in Fig 4.20 as follows:

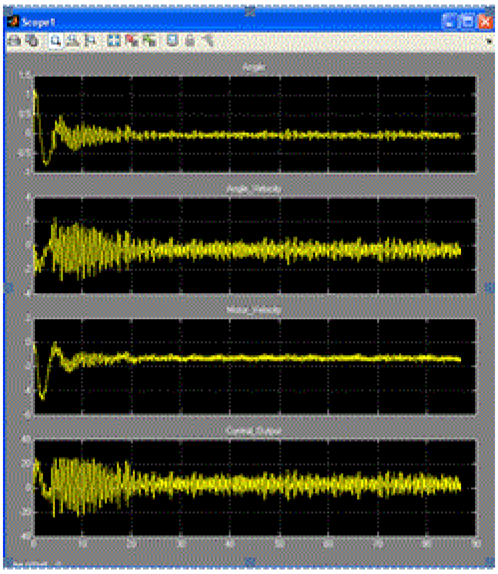


Fig 4.20. Response of self-balancing two-wheeled bicycle when it carries the load

- Operation of self-balancing two-wheeled bicycle when it carries the load is shown in Fig 4.22 as follows:

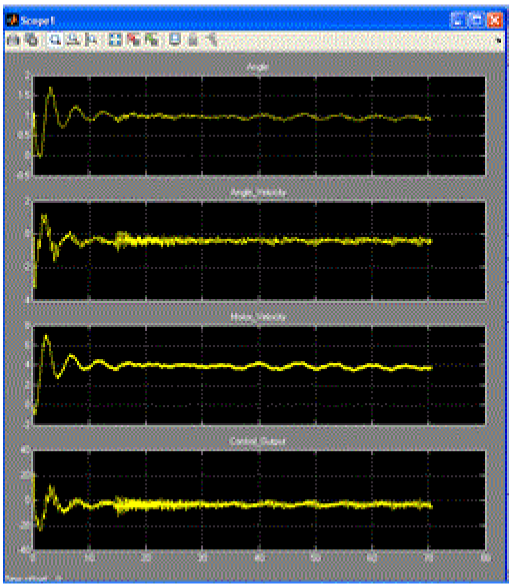


Fig 4.22. Response of self-balancing two-wheeled bicycle when it carries the eccentric load

- After experimentally controlling two-wheeled bicycle by Matlab – Simulink through a direct connection to the Adruno motherboard, the author wrote code controller and loaded directly into the Adruno motherboard to self-operating bicycle (Appendix 12 ***ABRB_alone.ino***). Experimental results of self-balancing two-wheeled bicycle is shown in Fig 4.23 as follows:

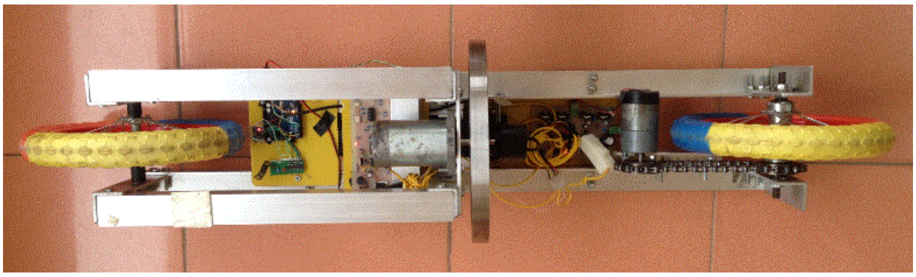


Fig 4.23. Photo of self-balancing two-wheeled bicycle when it doesn't carry load

Remarks of the results: Experimental result when loaded code controller in the Adruno motherboard to self-operating bicycle shows that: two-wheeled bicycle can stable balancing when bicycle stops, bicycle moves straight.

4.3. Conclusion of chapter 4

In chapter 4, author gained some following contents:

1. Building and connecting experimental control system of self-balancing two-wheeled bicycle (bicyle model is built in a laboratory) including two-wheeled bicycle model and a computer installed the Matlab - Simulink software.

2. Experimentally controlling two-wheeled bicycle (bicyle model is built in a laboratory) using order reduction robust controller through a direct connection to the Adruno motherboard on bicyle model with Matlab - Simulink software on computer showed that bicycle can stay balance when it doesn't carry load, it is affected by an external force, it caries load (balancing both sides), it caries eccentric load. Experimenting when loaded code controller into the Adruno motherboard showed that bicycle can self-operating (not connecting to the Adruno motherboard on bicyle model with Matlab - Simulink software on computer) and ensure stable balance.

CONCLUSIONS AND RECOMMENDATIONS

1. Conclusion

The dissertation gave the following results:

1. Building a new model order reduction algorithm for linear stable system. In which, the author gave three evaluation criterias (measuring) the importance (dominance) of the poles and built new algorithms for stable system by transforming the **A** matrix of the original system to upper - triangle matrix on which the poles are evaluated and arranged in descending important (dominant) properties on the main diagonal of the upper – triangle matrix, by this way, the author can preserve/ensure the dominant poles of original system in reduced order system as well as

obtain small error reduction. This algorithm is also applied for unstable system follow indirect method. Parallel to that, the author also provided a new theorem that determined the upper bound formula of order reduction error, 5 new lemmas and illustrative examples (reduced digital filter model, reduced CD player models, and reduced higher order unstable linear model) to prove the correctness and the effectiveness of the new algorithm.

2. Determining the upper bound formula of order reduction error of extend balance truncation algorithm in Zilochian study (1991) to reduce order of unstable system which help to easier evaluate order reduction error as well as help to perform automatic order reduction based on the upper bound formula of order reduction error. At the same time, the author also provided 2 new theorems that an example to prove the correctness and the effectiveness of the new algorithm.

3. Illustrating the effectiveness of two algorithms by reducing high order controller showed results as below

+ For the first problem: Reduced order of high-order robust controller in robust load angle control problem of synchronous generators (Trung research (2012)) showed that the 4^{th}-order controller can replace the original 28^{th}-order one but thequality of controller is still guaranteed. Beside, the order of the 4^{th}-order controllers in this study is lower than the one of controller (6^{th}-order) in Trung research (2012). The results have been verified through computer simulations.

+ For the second problem: Reduced order of high-order robust controller in balance control of two-wheeled bicycle (bicyle model is built in a laboratory) problem showed that the 5^{th},4^{th}-order controller can replace the high-order robust controller (30^{th}-order) but still the quality of controller is still guaranteed. In naturally, applying the 5^{th}, 4^{th} -order reduction controller will make programming code more simple, increase the response speed of system, ensure requirement on real-time control. The results have been verified through computer simulations and experimented on two-wheeled bicycle model.

2. Recommendation and future works

1. Study to apply two new algorithms to solve model order reduction problems in control field such as reduce high-order object model, ...

2. Building toolbox of two new algorithms in Matllab – Simulink